

1-1  
~~1-5~~ Speed of Sound and Mach Number

sound : a pressure wave of infinitesimal strength

infinitesimal p. change  $\Rightarrow$  reversible  
 little time for heat transfer  $\Rightarrow$  adiabatic  
 (and small  $\Delta T$ )  $\Rightarrow$  isentropic

$\Rightarrow a^2 = \left(\frac{\partial p}{\partial \rho}\right)_s$  ;  $a$ : sonic speed ;  $s$ : entropy

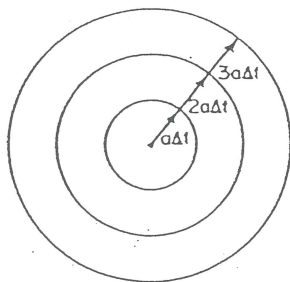
or  $a = \sqrt{\frac{E_v}{\rho}}$  or  $a = \sqrt{\frac{\kappa}{\rho}}$  ; in terms of bulk modulus,  
 $E_v$  (or  $\kappa$ ) =  $\frac{dp}{d\rho/\rho} = \rho \frac{dp}{d\rho}$

For an ideal gas :

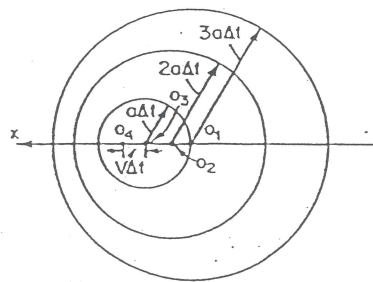
$\Rightarrow a = \sqrt{\frac{\gamma p}{\rho}} = \sqrt{\gamma RT}$  ;  $\gamma \equiv \frac{C_p}{C_v}$  = specific heat ratio

Define Mach Number,  $M \equiv \frac{V}{a}$

$\Rightarrow$

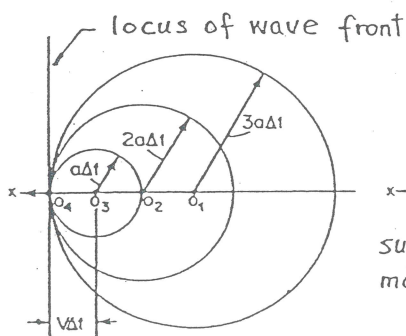


(a)  $V=0$  : stationary source

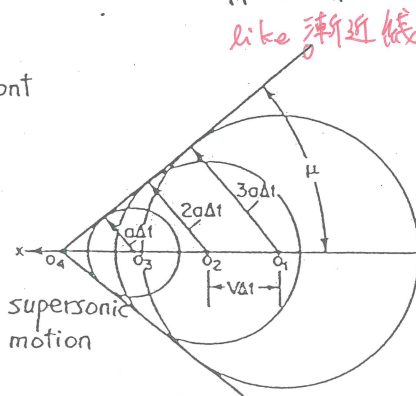


(b)  $V < a$  : Doppler Effect

Doppler Effect :  
 A stationary observer would hear more peaks per unit time as the source approaches than after it passes.

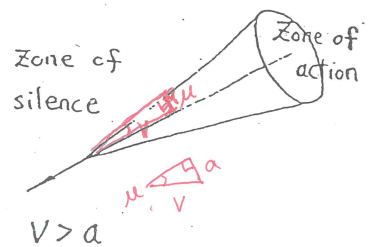


(c)  $V=a$



(d)  $V > a$

Mach Cone :



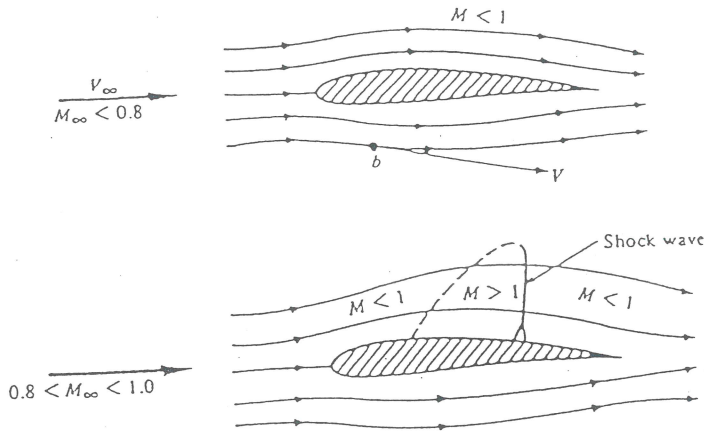
$\sin \mu = \frac{a}{V} = \frac{1}{M}$   
 Mach angle  $\mu = \sin^{-1}\left(\frac{1}{M}\right)$

Sonic wave 被限於 Zone of action

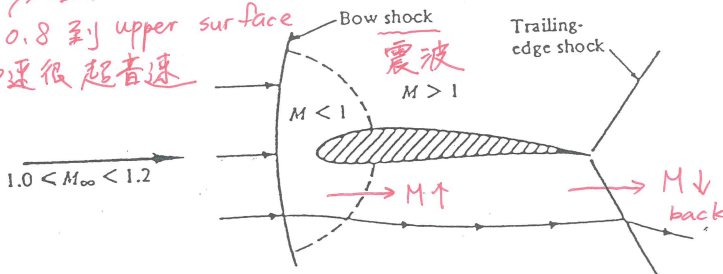
Navier-Stokes 從橢圓  $\rightarrow$  hyperbolic 雙曲綫

~~1-5-2~~ Classification of Compressible Flows

The compressible flows can be classified based on Mach number,  $M$ .



→ 低於 0.8 不太會超過音速  
0.8 到 upper surface  
加速後超音速



(1) Subsonic Flow

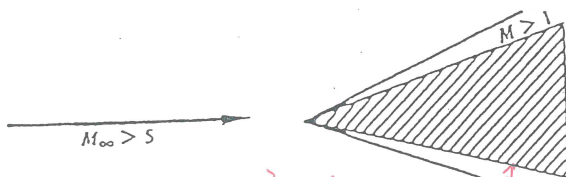
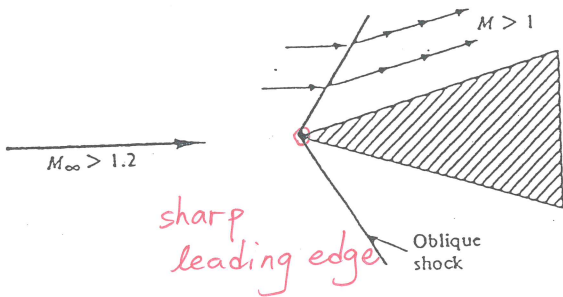
- (a)  $M < 1$  at every point
- (b) Smooth streamlines and continuously varying properties.
- (c) Streamlines deflect far upstream of the body.
- (d)  $M_\infty \leq 0.8$  for airplane aerodynamics.

(2) Transonic Flow

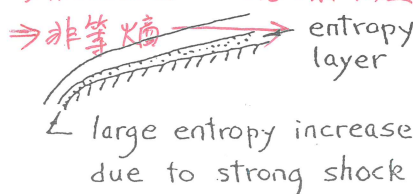
- (a) subsonic + supersonic.
- (b)  $0.8 < M_\infty < 1.2$ .
- (c) Subsonic region exists after bow shock.

(3) Supersonic Flow

- (a)  $M > 1$  everywhere.
- (b) A straight, oblique shock wave is attached to the sharp nose of the wedge.
- (c) Free stream is unaware of the presence of the body.



變化很大  
→ irreversible  
→ 非等熵  
截面積極小  
→ 摩擦 + 高溫 → 解離



(4) Hypersonic Flow

- (a)  $M_\infty > 5$ .
- (b) Thin shock layer.
- (c) High-temperature shock layer.
- (d) Entropy layer.

TABLE B.2. ISENTROPIC FLOW (Concluded)  
Perfect Gas,  $k = 1.4$ 

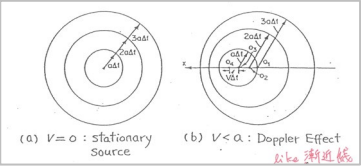
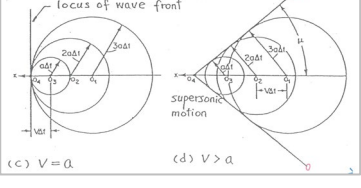
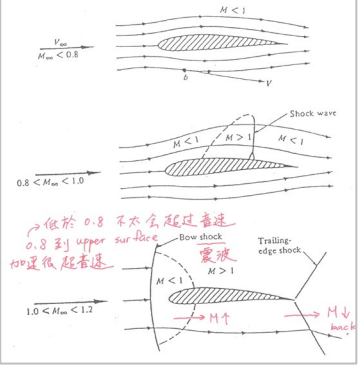
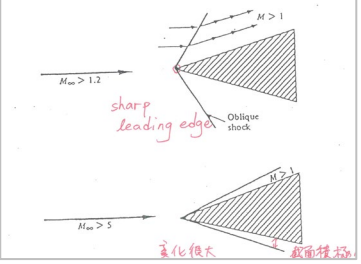
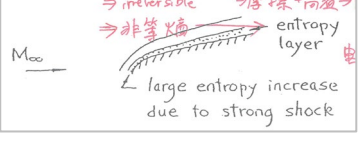
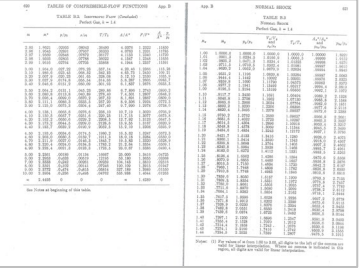
M	M*	$p/p_0$	$\rho/\rho_0$	$T/T_0$	$A/A^*$	$F/F^*$	$\frac{A}{A^*} \frac{p}{p_0}$
2.95	1.9521	.02935	.08043	.36490	4.0376	1.2322	1.1850
2.96	1.9545	.02891	.07957	.36333	4.0763	1.2331	1.1750
2.97	1.9569	.02848	.07872	.36177	4.1153	1.2340	1.1720
2.98	1.9593	.02805	.07788	.36022	4.1547	1.2348	1.1656
2.99	1.9616	.02764	.07705	.35868	4.1944	1.2357	1.1591
3.00	1.9640	.02722	.07623	.35714	4.2346	1.2366	1.1528
3.10	1.9866	.02345	.06152	.34223	4.6573	1.2450	1.1092
3.20	2.0079	.02023	.05370	.32808	5.1210	1.2530	1.0359
3.30	2.0279	.01748	.04554	.31466	5.6287	1.2605	0.9837
3.40	2.0466	.01512	.03509	.30193	6.1837	1.2676	0.9353
3.50	2.0642	.01311	.02423	.28986	6.7896	1.2743	0.8902
3.60	2.0808	.01138	.01839	.27840	7.4501	1.2807	0.8482
3.70	2.0964	.00990	.01370	.26752	8.1691	1.2867	0.8090
3.80	2.1111	.00863	.00935	.25720	8.9506	1.2924	0.7723
3.90	2.1250	.00753	.00704	.24740	9.7990	1.2978	0.7380
4.00	2.1381	.00658	.00576	.23810	10.719	1.3029	0.7059
4.10	2.1505	.00577	.00451	.22925	11.715	1.3077	0.6758
4.20	2.1622	.00506	.00329	.22085	12.792	1.3123	0.6475
4.30	2.1732	.00445	.02090	.21286	13.955	1.3167	0.6209
4.40	2.1837	.00392	.01909	.20525	15.210	1.3208	0.5959
4.50	2.1936	.00346	.01745	.19802	16.562	1.3247	0.5723
4.60	2.2030	.00305	.01597	.19113	18.018	1.3284	0.5500
4.70	2.2119	.00270	.01463	.18467	19.583	1.3320	0.5289
4.80	2.2204	.00240	.01333	.17832	21.264	1.3354	0.5091
4.90	2.2284	.00213	.01233	.17235	23.067	1.3386	0.4904
5.00	2.2361	.00189	.01134	.16667	25.000	1.3416	0.4725
6.00	2.2953	.00633	.00519	.12195	53.180	1.3655	0.3368
7.00	2.3333	.03242	.00261	.09259	104.143	1.3810	0.2516
8.00	2.3591	.03102	.00141	.07246	190.109	1.3915	0.1947
9.00	2.3772	.04474	.00315	.05314	327.189	1.3989	0.1550
10.00	2.3904	.04236	.03495	.04762	535.938	1.4044	0.1263
$\infty$	2.4495	0	0	0	$\infty$	1.4289	0

See Notes at beginning of this table.

TABLE B.3  
NORMAL SHOCK  
Perfect Gas,  $k = 1.4$ 

$M_x$	$M_y$	$p_y/p_x$	$\frac{V_x}{V_y}$ and $\frac{\rho_y}{\rho_x}$	$T_y/T_x$	$\frac{A_x^*}{A_y^*}$ and $\frac{p_{0y}}{p_{0x}}$	$\frac{p_{0y}}{p_x}$
1.00	1.0000, 0	1.0000, 0	1.0000, 0	1.0000, 0	1.00000	1.8929
1.01	.9901, 2	1.0234, 5	1.0166, 9	1.0066, 5	.99999	1.9152
1.02	.9805, 2	1.0471, 3	1.0334, 4	1.01325	.99998	1.9379
1.03	.9711, 5	1.0710, 5	1.0502, 4	1.01981	.99997	1.9610
1.04	.9620, 2	1.0952, 0	1.0670, 9	1.02634	.99994	1.9845
1.05	.9531, 2	1.1196	1.0839, 8	1.03284	.99987	2.0083
1.06	.9444, 4	1.1442	1.10092	1.03931	.99976	2.0325
1.07	.9359, 8	1.1690	1.11790	1.04575	.99962	2.0570
1.08	.9277, 2	1.1941	1.13492	1.05217	.99944	2.0819
1.09	.9196, 5	1.2194	1.15199	1.05856	.99921	2.1072
1.10	.9117, 7	1.2450	1.1691	1.06494	.99899	2.1328
1.11	.9040, 8	1.2708	1.1862	1.07130	.99878	2.1588
1.12	.8965, 6	1.2968	1.2034	1.07764	.99850	2.1851
1.13	.8892, 2	1.3230	1.2206	1.08396	.99776	2.2118
1.14	.8820, 4	1.3495	1.2378	1.09027	.9972, 6	2.2388
1.15	.8750, 2	1.3762	1.2550	1.09657	.9966, 9	2.2661
1.16	.8681, 6	1.4032	1.2723	1.10287	.9960, 5	2.2937
1.17	.8614, 5	1.4304	1.2896	1.10916	.9953, 4	2.3217
1.18	.8548, 8	1.4578	1.3069	1.11544	.9945, 4	2.3499
1.19	.8484, 6	1.4854	1.3243	1.12172	.9937, 1	2.3786
1.20	.8421, 7	1.5133	1.3416	1.1280	.9928, 0	2.4075
1.21	.8360, 1	1.5414	1.3590	1.1343	.9918, 0	2.4367
1.22	.8299, 8	1.5698	1.3764	1.1405	.9907, 3	2.4662
1.23	.8240, 8	1.5984	1.3938	1.1468	.9895, 7	2.4961
1.24	.8183, 0	1.6272	1.4112	1.1531	.9883, 5	2.5263
1.25	.8126, 4	1.6562	1.4286	1.1594	.9870, 6	2.5568
1.26	.8070, 9	1.6855	1.4460	1.1657	.9856, 8	2.5876
1.27	.8016, 5	1.7150	1.4634	1.1720	.9842, 2	2.6187
1.28	.7963, 1	1.7448	1.4808	1.1782	.9826, 8	2.6500
1.29	.7910, 8	1.7748	1.4983	1.1846	.9810, 6	2.6816
1.30	.7859, 6	1.8050	1.5157	1.1909	.9793, 5	2.7136
1.31	.7809, 3	1.8354	1.5331	1.1972	.9775, 8	2.7457
1.32	.7760, 0	1.8661	1.5505	1.2035	.9757, 4	2.7783
1.33	.7711, 6	1.8970	1.5680	1.2099	.9738, 2	2.8112
1.34	.7664, 1	1.9282	1.5854	1.2162	.9718, 1	2.8444
1.35	.7617, 5	1.9596	1.6028	1.2226	.9697, 2	2.8778
1.36	.7571, 8	1.9912	1.6202	1.2290	.9675, 6	2.9115
1.37	.7526, 9	2.0230	1.6376	1.2354	.9653, 4	2.9455
1.38	.7482, 8	2.0551	1.6550	1.2418	.9630, 4	2.9798
1.39	.7439, 6	2.0874	1.6723	1.2482	.9606, 5	3.0144
1.40	.7397, 1	2.1200	1.6896	1.2547	.9581, 9	3.0493
1.41	.7355, 4	2.1528	1.7070	1.2612	.9556, 6	3.0844
1.42	.7314, 4	2.1858	1.7243	1.2676	.9530, 6	3.1198
1.43	.7274, 1	2.2190	1.7416	1.2742	.9503, 9	3.1555
1.44	.7234, 5	2.2525	1.7589	1.2807	.9476, 5	3.1915

Notes: (1) For values of M from 1.00 to 3.00, all digits to the left of the comma are valid for linear interpolation. Where no comma is indicated in this region, all digits are valid for linear interpolation.

NO.	圖片	來源
1	 <p>(a) <math>V = 0</math>: stationary source (b) <math>V &lt; a</math>: Doppler Effect <i>紅字: 漸近線</i></p>	<p>John D. Anderson Jr., Modern Compressible Flow, Tata McGrawhill India Pvt Ltd; 3rd edition (January 1, 2012), p. 132.</p> <p>This work is used subject to the fair use doctrine of Article 52, 65 Taiwan Copyright Act.</p>
2	 <p>(c) <math>V = a</math> (d) <math>V &gt; a</math> supersonic motion</p>	<p>John D. Anderson Jr., Modern Compressible Flow, Tata McGrawhill India Pvt Ltd; 3rd edition (January 1, 2012), p. 132.</p> <p>This work is used subject to the fair use doctrine of Article 52, 65 Taiwan Copyright Act.</p>
3	 <p><math>M &lt; 1</math> <math>M &lt; 1</math> <math>0.8 &lt; M_{\infty} &lt; 1.0</math> <math>1.0 &lt; M_{\infty} &lt; 1.2</math> Shock wave Bow shock Trailing edge shock <i>紅字: 低於 0.8 不會起超音速, 0.8 到 upper surface 加速後起超音速</i></p>	<p>John D. Anderson Jr., Modern Compressible Flow, Tata McGrawhill India Pvt Ltd; 3rd edition (January 1, 2012), p. 16.</p> <p>This work is used subject to the fair use doctrine of Article 52, 65 Taiwan Copyright Act.</p>
4	 <p><math>M_{\infty} &gt; 1.2</math> <math>M_{\infty} &gt; 5</math> sharp leading edge Oblique shock <i>紅字: 變化很大, 數值極大</i></p>	<p>John D. Anderson Jr., Modern Compressible Flow, Tata McGrawhill India Pvt Ltd; 3rd edition (January 1, 2012), p. 16.</p> <p>This work is used subject to the fair use doctrine of Article 52, 65 Taiwan Copyright Act.</p>
5	 <p><math>M_{\infty}</math> entropy layer large entropy increase due to strong shock <i>紅字: irreversible, 非等熵, 熵層, 厚, 熵增加</i></p>	<p>John D. Anderson Jr., Introduction to Flight, McGraw Hill; 8th edition (February 19, 2015), p. 821.</p> <p>This work is used subject to the fair use doctrine of Article 52, 65 Taiwan Copyright Act.</p>
6		<p>Ascher H. Shapiro, The Dynamics and Thermodynamics of Compressible Fluid Flow, Vol. 1, Wiley; 1st edition (January 16, 1991).</p> <p>This work is used subject to the fair use doctrine of Article 52, 65 Taiwan Copyright Act.</p>