19 The Nucleus

Protons
Neutrons
Nucleons

Total number of nucleons: mass number $^{238}\text{U}$
Number of protons: atomic number $^{92}\text{U}$

Isotopes: identical atomic numbers different mass numbers
same chemical properties different nuclear properties

※ Nuclear stability and radioactive decay

Radioactivity: reflects kinetic stability

◎ Types of radioactive decay

- Frequently accompanied by $\gamma$-ray emission
  high energy electromagnetic radiation
- $\alpha$-particle production
  $^{238}_{92}\text{U} \rightarrow \ ^{4}_{2}\text{He} + \ ^{234}_{90}\text{Th}$
  $\alpha$ particle
- $\beta$-particle production
  $^{234}_{90}\text{Th} \rightarrow \ ^{234}_{91}\text{Pa} + \ ^{0}_{-1}\text{e}$
  $\beta$ particle
  ($\beta$-particle production frequently accompanied by $\gamma$-ray emission)
- **Positron production**

\[ ^{22}_{11}\text{Na} \rightarrow ^{22}_{10}\text{Ne} + ^{0}_{1}\text{e} \]

\[(^1\text{p} \rightarrow ^1n + ^0\text{e})

\((^0\text{e} + ^0\text{e} \rightarrow 2^0\gamma)\]

- **Electron capture**

\[ ^{201}_{80}\text{Hg} + ^{0}_{-1}\text{e} \rightarrow ^{201}_{79}\text{Au} + ^{0}_0\gamma \]

- **Spontaneous fission (usually slow)**

\[ ^{254}_{98}\text{Cf} \rightarrow \text{lighter nuclides} + \text{neutrons} \]

- **Empirical rule of nuclear stability (prone to decay)**

  - With \( \geq 84 \) protons: less stable
  - Lighter nuclides:
    - \( n/p \) closer to 1 \( \rightarrow \) more stable
    - \( p \uparrow \rightarrow \text{ratio} \uparrow \)
  - Emit \( \beta^- \)-particle
  - Positron emission

\[ n \downarrow \quad p \uparrow \]

\[ n/p = 1 \]
• With even numbers of $p$s and $n$s $\rightarrow$ more stable

<table>
<thead>
<tr>
<th>stable isotopes</th>
<th>$p$</th>
<th>$n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>168</td>
<td>even</td>
<td>even</td>
</tr>
<tr>
<td>57</td>
<td>even</td>
<td>odd</td>
</tr>
<tr>
<td>50</td>
<td>odd</td>
<td>even</td>
</tr>
<tr>
<td>4</td>
<td>odd</td>
<td>odd</td>
</tr>
</tbody>
</table>

• With magic numbers of $p$s or $n$s

$2, 8, 20, 50, 82, 126 \rightarrow$ more stable

<table>
<thead>
<tr>
<th>stable isotopes</th>
<th>$p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>18</td>
</tr>
<tr>
<td>2</td>
<td>19</td>
</tr>
<tr>
<td>5</td>
<td>20 (Ca)</td>
</tr>
<tr>
<td>1</td>
<td>21</td>
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</tbody>
</table>

© Decay series
emission in a series

$^{238}_{92}U \rightarrow ^{234}_{90}Th \rightarrow ^{234}_{91}Pa \rightarrow ^{206}_{82}Pb$
The kinetics

Decay is an unimolecular process

Rate = \(- \frac{dN}{dt} = kN\) \((N \text{ for number of nuclides})\)

\[\ln\left(\frac{N}{N_0}\right) = -kt\]

\[t_{1/2} = \frac{0.693}{k}\]

A characteristic value

Unaffected by \(T, P\) or chemical form

No way to stop!!

Ex. \(^{238}\text{U}\) half-life: \(4.5 \times 10^9\) yr

Dating

C-14 method

\(^{14}_7\text{N} + ^{1}_0\text{n} \rightarrow ^{14}_6\text{C} + ^{1}_1\text{H}\)

\(^{14}_6\text{C} \rightarrow ^{14}_7\text{N} + ^{0}_{-1}\text{e}\)

\(t_{1/2} = 5730\) yr

In atmosphere: reaching an equilibrium

\(^{14}_6\text{C}/^{12}_6\text{C}\) remains constant

\(*\text{CO}_2 \rightarrow *\text{Organic molecule in plants} \rightarrow *\text{animals}\)

↓

From \(^{14}_6\text{C}/^{12}_6\text{C}\) to determine age \(\leftarrow\) decay \(\leftarrow\) died

(equilibrium stops)
Ex. If ratio is half that of atmosphere
→ 5370 yr old

Limitation: can not be older than 20000 yr
→ radioactivity too low to be accurate

Checked with tree growth: accurate within 10%

Oldest rock: $3 \times 10^9$ yr
Cooling time for earth surface: $1–1.5 \times 10^9$ yr
Age of earth at about: $4.0–4.5 \times 10^9$ yr

Ex. A rock with $0.115 \text{ mg } ^{206}\text{Pb}/1.000 \text{ mg } ^{238}\text{U}$
Assuming $^{206}\text{Pb}$ is coming from $^{238}\text{U}$
→ The original amount of $^{238}\text{U}$ was
$$0.115 \frac{\text{mg}}{206} \times 1.000 = 1.133 \text{ mg}$$

The amount decayed
For $^{238}\text{U}$  
$t_{1/2} = 4.5 \times 10^9$ yr = 0.693/$k$
$\rightarrow k = \frac{0.693}{4.5 \times 10^9 \text{ yr}^{-1}}$
$$\ln \frac{N}{N_o} = -kt = \ln \left(\frac{1.000}{1.133}\right) = -\left(\frac{0.693}{4.5 \times 10^9 \text{ yr}}\right)t$$
$\Rightarrow t = 8.1 \times 10^8$ yr
Nuclear transformations

The change of one element into another

1919 Rutherford

\[ ^{14}_2 \text{N} + ^{4}_2 \text{He} \rightarrow ^{17}_6 \text{O} + ^{1}_1 \text{H} \]

From Ra with high velocity

Particle accelerator
cyclotron, synchrotron, linear accelerator
to accelerate charged particle in order to overcome electrostatic repulsion

Cyclotron
High frequency alternating voltage in a vacuum chamber
With vertical magnetic field

✓ Neutron bombardment
no acceleration is necessary
more common for isotopes synthesis
neutral (generated from nuclear reactor)

\[ ^{238}_92 \text{U} + ^{1}_0 \text{n} \rightarrow ^{239}_92 \text{U} \rightarrow ^{239}_93 \text{Np} + ^{0}_1 \text{e} \]
\[ t_{1/2} = 23 \text{ min} \]

Elements with atomic # > 92 are synthesized via artificial transmutations – transuranium elements
※ Detection of radioactivity

- **Geiger counter**
  - Amplifier and counter
  - Window that can be penetrated by α, β or γ rays

Detect ionization by radiation

- **Scintillation counter**
  - ZnS: high-E radiation → fluorescence
  - (caused by electronic excitation)
  - Measurement

※ Thermodynamic stability

**Ex.** $^8_3\text{n} + ^8_1\text{H} \to ^{16}_{8}\text{O}$

Mass of reactants = $8(1.67493 \times 10^{-24}) + 8(1.67262 \times 10^{-24})$ g = $2.67804 \times 10^{-23}$ g

Mass of product = $2.65535 \times 10^{-23}$ g

Smaller by $0.1366$ g/mol

Einstein: mass energy conversion

$$E = mc^2$$ (c: speed of light = $3.00 \times 10^8$ m/s)

$$\Delta E = \Delta mc^2 = (-1.366 \times 10^{-4}$ kg/mol)(3.00 $\times 10^8$ m/s)$^2$$

= $-1.23 \times 10^{13}$ J/mol or $-2.04 \times 10^{-11}$ J per nucleus

= $-1.28 \times 10^{-12}$ J per nucleon

= $-8.00$ MeV/nucleon (1 MeV = $1.60 \times 10^{-13}$ J)

The reverse is the binding energy per nucleon
Ex. The binding E per nucleon for the $^4_2\text{He}$ nucleus

$AW(\text{He}) = 4.0026 \text{ amu}$  $\quad AW(\text{H}) = 1.0078 \text{ amu}$

Mass including electrons

Mass of $^4_2\text{He}$ nucleus = $AW(\text{He}) - 2m_e$

Mass of $^1_1\text{H}$ nucleus = $AW(\text{H}) - m_e$

$\Delta m = (4.0026 - 2m_e) - [2(1.0078 - m_e) + 2m_n]$

$\Delta E = \Delta mc^2 = (-0.0304 \text{ amu})(1.66 \times 10^{-27} \text{ Kg/amu})(3.00 \times 10^8 \text{ m/s})^2$

$= 4.54 \times 10^{-12} \text{ J/nucleus} \equiv 1.14 \times 10^{-12} \text{ J/nucleon}$

※ Nuclear fission and fusion
More neutrons are produced – may be explosive.

Further fission

Cover 200 isotopes, 35 elements

Produce 2.4 neutron in average

Size of the sample

- Too small: neutron escapes before striking a nucleus – subcritical
- Too large: neutrons are completely consumed – super critical
- In between: chain reactions keep at a constant rate (one for one) – critical

Nuclear reactor

keep a self-sustaining chain reaction

\( ^{235}\text{U} \) enriched to 3% (natural abundance: 0.724)

\( \rightarrow \) \( ^{236}\text{U} \) pellets in Zr or stainless steel tubes

Control rods: made of Cd or B to absorb neutrons

\( ^{10}\text{B} + ^{1}\text{n} \rightarrow ^{7}\text{Li} + ^{4}\text{He} \)

Water used as moderator (slow down neutrons but not reacting)
Nuclear power $\rightarrow$ thermal E $\rightarrow$ steam $\rightarrow$ steam engine $\downarrow$

biproducts $\rightarrow$ reprocess $\rightarrow$ reusable fuel $\downarrow$

electricity generation

waste such as $^{90}\text{Sr}$ $t_{1/2} = 28.8$ yr

$^{239}\text{Pu}$ $t_{1/2} = 2400$ yr

© Fusion

In the sun $^{1}\text{H} + ^{1}\text{H} \rightarrow ^{2}\text{H} + ^{0}\text{e}$

$^{1}\text{H} + ^{2}\text{H} \rightarrow ^{3}\text{He}$

$^{3}\text{He} + ^{3}\text{He} \rightarrow ^{4}\text{He} + ^{2}\text{H}$

$^{3}\text{He} + ^{1}\text{H} \rightarrow ^{4}\text{He} + ^{0}\text{e}$

Products are generally not radioactive

High E necessary: to overcome repulsions occurs at high T
Effect of radiation

Somatic damage: damage to the organism itself
Genetic damage: damage to the genetic machinery

Biological effects
• The energy
  measured in rads (radiation absorbed dose)
  \[ 1 \text{ rad} = 10^{-2} \text{ J/kg tissue} \]
• The penetrating ability
  \( \alpha \) – stops at skin
  \( \beta \) – down 1 cm
  \( \gamma \) – highly penetrating (the most dangerous)
• The ionizing ability
  \( \alpha \) – very damaging
• The chemical property of the source
  Overall effect:
  \[ \text{rem} = \text{rad} \times \text{RBE} \]