

邏輯作業(Unit 12)

系級：_____

姓名：_____

學號：_____

日期：_____

請以自然演繹法證明下列語法序列。

1. $(\exists x)(Px \wedge Qx), (\forall y)(Py \rightarrow Ry) \vdash (\exists z)(Qz \wedge Rz)$

2. $(\forall x)\neg(Lx \wedge \neg Mx), (\forall y)(Ny \rightarrow Ly), (\forall z)(Kz \rightarrow Nz) \vdash (\forall x)(Kx \rightarrow Mx)$

3. $(\forall x)\neg(Wx \wedge Sx), \neg(\exists x)(Tx \wedge Rx), (\forall x)(\neg Wx \rightarrow Tx) \vdash (\exists x)\neg(Sx \wedge Rx)$

4. $(\forall x)(\forall y)(Pxy \rightarrow Qx), (\exists x)(\exists y)Pxy \vdash (\exists x)Qx$

5. $(\forall x)(\exists y)(Ax \rightarrow Bxy), (\exists x)Ax \vdash (\exists x)(\exists y)Bxy$

邏輯作業(Unit 12)

解答

系級：_____

姓名：_____

學號：_____

日期：_____

請以自然演繹法證明下列語法序列。

$$1. (\exists x)(Px \wedge Qx), (\forall y)(Py \rightarrow Ry) \vdash (\exists z)(Qz \wedge Rz)$$

1.	$(\exists x)(Px \wedge Qx)$	Pr
2.	$(\forall y)(Py \rightarrow Ry)$	Pr
3.	$Pa \wedge Qa$	1, EI
4.	Pa	3, Simp
5.	$Pa \rightarrow Ra$	2, UI
6.	Ra	4, 5, MP
7.	Qa	3, Simp
8.	$Qa \wedge Ra$	6, 7, Conj
9.	$(\exists z)(Qz \wedge Rz)$	8, EG

$$2. (\forall x)\neg(Lx \wedge \neg Mx), (\forall y)(Ny \rightarrow Ly), (\forall z)(Kz \rightarrow Nz) \vdash (\forall x)(Kx \rightarrow Mx)$$

1.	$(\forall x)\neg(Lx \wedge \neg Mx)$	Pr
2.	$(\forall y)(Ny \rightarrow Ly)$	Pr
3.	$(\forall z)(Kz \rightarrow Nz)$	Pr
4.	$Ka \rightarrow Na$	3, UI
5.	$Na \rightarrow La$	2, UI
6.	$\neg(La \wedge \neg Ma)$	1, UI
7.	$\neg La \vee \neg \neg Ma$	6, DeM
8.	$\neg La \vee Ma$	7, DN
9.	$La \rightarrow Ma$	8, Impl
10.	$Ka \rightarrow La$	4, 5, HS
11.	$Ka \rightarrow Ma$	9, 10, HS
12.	$(\forall x)(Kx \rightarrow Mx)$	11, UG

3. $(\forall x)\neg(Wx \wedge Sx), \neg(\exists x)(Tx \wedge Rx), (\forall x)(\neg Wx \rightarrow Tx) \vdash (\exists x)\neg(Sx \wedge Rx)$

1.	$(\forall x)\neg(Wx \wedge Sx)$	Pr
2.	$\neg(\exists x)(Tx \wedge Rx)$	Pr
3.	$(\forall x)(\neg Wx \rightarrow Tx)$	Pr
4.	$(\forall x)\neg(Tx \wedge Rx)$	2, QN
5.	$\neg(Ta \wedge Ra)$	4, UI
6.	$\neg Ta \vee \neg Ra$	5, DeM
7.	$Ta \rightarrow \neg Ra$	6, Impl
8.	$\neg Wa \rightarrow Ta$	3, UI
9.	$\neg(Wa \wedge Sa)$	1, UI
10.	$\neg Wa \vee \neg Sa$	9, DeM
11.	$\neg Sa \vee \neg Wa$	10, Comm
12.	$Sa \rightarrow \neg Wa$	11, Impl
13.	$Sa \rightarrow Ta$	8, 12, HS
14.	$Sa \rightarrow \neg Ra$	7, 13, HS
15.	$\neg Sa \vee \neg Ra$	14, Impl
16.	$\neg(Sa \wedge Ra)$	15, DeM
17.	$(\exists x)\neg(Sx \wedge Rx)$	16, EG

4. $(\forall x)(\forall y)(Pxy \rightarrow Qx), (\exists x)(\exists y)Pxy \vdash (\exists x)Qx$

1.	$(\forall x)(\forall y)(Pxy \rightarrow Qx)$	Pr
2.	$(\exists x)(\exists y)Pxy$	Pr
3.	$(\exists y)Pxy$	2, EI
4.	Pab	3, EI
5.	$(\forall y)(Pxy \rightarrow Qa)$	1, UI
6.	$Pab \rightarrow Qa$	5, UI
7.	Qa	4, 6, MP
8.	$(\exists x)Qx$	7, EG

5. $(\forall x)(\exists y)(Ax \rightarrow Bxy), (\exists x)Ax \vdash (\exists x)(\exists y)Bxy$

1.	$(\forall x)(\exists y)(Ax \rightarrow Bxy)$	Pr
2.	$(\exists x)Ax$	Pr
3.	Aa	2, EI
4.	$(\exists y)(Aa \rightarrow Bay)$	1, UI
5.	$Aa \rightarrow Bab$	4, EI
6.	Bab	3, 5, MP
7.	$(\exists y)Bay$	6, EG
8.	$(\exists x)(\exists y)Bxy$	7, EG