※ Electromagnetic radiation (Maxwell, 1864) (nature of light) Composed of oscillating perpendicular electric field and magnetic field


$$
\begin{gathered}
\text { If } \Delta t=1 \mathrm{sec} \Rightarrow 1 \text { cycle per second }=1 \mathrm{~Hz}=1 \mathrm{~s}^{-1} \\
\lambda v=\mathrm{c} \\
\\
\\
\\
c=3.0 \times 10^{8} \mathrm{~m} / \mathrm{s}
\end{gathered}
$$

Black body radiation（or box with a pinhole）


ULTRAVIOLET CATASTROPHE

## 1901 Max Planck

Postulate：the energies are discrete and are integers of $h v$

$$
\begin{aligned}
& h=6.626 \times 10^{-34} \mathrm{Js} \\
& \downarrow \text { Planck constant }
\end{aligned}
$$

\[

\]

Energies are gained or lost in $n h v$
$\Delta E=n h v \quad n$ is an integer $v$ ：freq．of radiation absorbed or emitted
$\Rightarrow$ Now the black body radiation profile can be derived
$\Rightarrow$ Meaning：
The energy of light is quantized
Energy exchanged in whole＂quanta＂（quantum是複數）Photoelectric phenomenon
1887 Hertz
Light strikes on metal $\quad \Rightarrow \quad e^{-}$emitted
Lenard
A minimum $E$ required $\left(v_{0}\right)$

$$
\begin{array}{ll}
v<v_{0} & \text { no } e^{-} \\
v>v_{0} & \text { yes }
\end{array}
$$

Light intensity increases the number of $\mathrm{e}^{-}$ but not the $E$ of $\mathrm{e}^{-}$

1905 Einstein (1879-1955)
$\checkmark$ Electromagnetic radiation is quantized

$$
E_{\text {photon }}=h \nu=h \mathrm{c} / \lambda
$$

Predicted:

$$
h v-h v_{\mathrm{o}}=K E_{\mathrm{e}^{-}}=1 / 2 m v^{2} \quad \text { Unrelated to }
$$ light intensity

Work function (P): The amount of work that the $\mathrm{e}^{-}$must produce on leaving the body

Confirmed by Hughes, Richardson and Compton (1912) and Millikan (1916)
$\checkmark$ Photon has mass (not a rest mass)

$$
m=\frac{E}{\mathrm{c}^{2}}=\frac{h}{\lambda c} \quad \text { or } E=m c^{2}
$$

1922 Compton: Confirmed by collision of X-rays and $\mathrm{e}^{-}$
$\Rightarrow \quad$ Light has dual nature: wave and particle

1924 de Broglie (1892-1987)
Particle also has wave nature

$$
\left.m=\frac{h}{\lambda v} \quad \text { (cf: } m=\frac{h}{\lambda c}\right)
$$

de Broglie equation: $\lambda=\frac{h}{m v}$
Ex. $\quad m_{\mathrm{e}}=9.11 \times 10^{-31} \mathrm{~kg}$
If traveling at a speed of $1.0 \times 10^{7} \mathrm{~m} / \mathrm{s}$

$$
\begin{aligned}
\lambda=\frac{h}{m v} & =\frac{6.626 \times 10^{-34} \mathrm{kgm}^{2} / \mathrm{s}}{\left(9.11 \times 10^{-31} \mathrm{~kg}\right)\left(1.0 \times 10^{7} \mathrm{~m} / \mathrm{s}\right)} \\
& =7.3 \times 10^{-11} \mathrm{~m}
\end{aligned}
$$

In the range of $X$-ray
Same as the spacing between atoms in a crystal

## 1927 Davison and Germer (Bell lab)

A beam of $e^{-}$hitting a nickel crystal
$\Rightarrow$ diffraction occurs
Verified the wave properties of $\mathrm{e}^{-}$

Conclusion
All matter exhibits both particulate and wave properties
Larger particle $\longleftrightarrow$ Photons
More particulate-like More wave-like

※ The Bohr model
NTU ©hemistry

1913 Bohr (1885-1962)
nucleus (with atomic number $Z$ )
circles around the nucleus
(in a velocity of v )

Problems of classical physics:
accelerating charged particle $\Rightarrow$ radiate energy
$\Rightarrow$ lose E
$\Rightarrow$ drops into nucleus


Bohr's model was based on experimental results
Proposed the angular momentum of the electron could occur only in certain increment

$$
\begin{aligned}
& m v r=n \hbar \quad(\hbar=h / 2 \pi \quad n=1,2,3 \ldots .) \\
\Rightarrow & v^{2}=\frac{n^{2} \hbar^{2}}{m^{2} r^{2}} \quad \Rightarrow \quad r=\frac{Z e^{2}}{m v^{2}}=\frac{Z e^{2}}{m} \cdot \frac{m^{2} r^{2}}{n^{2} \hbar^{2}}=\frac{Z e^{2} m r^{2}}{n^{2} \hbar^{2}}
\end{aligned}
$$

$\Rightarrow E=-\frac{Z e^{2}}{2} \frac{1}{r}=-\frac{Z e^{2}}{2} \frac{Z e^{2} m}{n^{2} \hbar^{2}} \Rightarrow E=-2.178 \times 10^{-18}\left(Z^{2} / \mathrm{n}^{2}\right) \mathrm{J}$

```
M
    One mole with n=1
    =>E=-13.6 eV = -1310 kJ per mol
    n=\infty g E=0 ( a reference point)
        n\downarrow E\downarrow
    From n=6 }->\mathrm{ n=1
    E
    \DeltaE=\mp@subsup{E}{n=1}{}-\mp@subsup{E}{n=6}{}}={-2.178\times1\mp@subsup{0}{}{-18}[(1/\mp@subsup{1}{}{2})-(1/\mp@subsup{6}{}{2})
```

$$
\begin{gathered}
\Delta E=h \frac{c}{\lambda} \Rightarrow \lambda=\frac{h c}{\Delta E} \\
\lambda=\frac{\left(6.626 \times 10^{-34}\right)\left(2.9979 \times 10^{8}\right)}{2.118 \times 10^{-18}}=9.379 \times 10^{-8} \mathrm{~m}
\end{gathered}
$$

Ex. $n=1 \rightarrow n=2$
$\Delta E=-2.178 \times 10^{-18}\left(\frac{1}{2} 2^{2}-1\right)$
$=1.634 \times 10^{-18} \mathrm{~J}$
$\lambda=1.216 \times 10^{-7} \mathrm{~m}=121.6 \times 10^{-9} \mathrm{~m}=121.6 \mathrm{~nm}$

For H：$n=5 \rightarrow n=2$ blue
$n=4 \rightarrow n=2$ green
$n=3 \rightarrow n=2$ red
Overall：$\Delta E=E_{\text {final }}-E_{\text {initial }}=-2.178 \times 10^{-18} \mathrm{~J}\left(\frac{1}{n_{f}^{2}}-\frac{1}{n^{2}}\right)$
$n=1 \Rightarrow$ ground state
From $n=1 \rightarrow n=\infty \Rightarrow$ remove $\mathrm{e}^{-}$from the ground state $\Delta E=-2.178 \times 10^{-18}\left(\frac{1}{\infty} \infty^{2}-1\right)=2.178 \times 10^{-18} \mathrm{~J}$Problems with Bohr＇s model
Only works for H atom
$\Rightarrow \quad$ can not be correct
The idea of quantization is influential
※ The quantum mechanical description of the atom

1925－1926
Heisenberg，de Broglie，Schrödinger $\Rightarrow$ Wave mechanics or quantum mechanics

## The Nobel Prize in Physics

1911 Wilhelm Wien "for his discoveries regarding the laws governing the radiation of heat"
1918 Max Karl Ernst Ludwig Planck "in recognition of the services he rendered to the advancement of Physics by his discovery of energy quanta"
1921 Albert Einstein "for his services to Theoretical Physics, and especially for his discovery of the law of the photoelectric effect"
1922 Niels Henrik David Bohr "for his services in the investigation of the structure of atoms and of the radiation emanating from them"
1929 Prince Louis-Victor Pierre Raymond de Broglie "for his discovery of the wave nature of electrons"
1932 Werner Karl Heisenberg "for the creation of quantum mechanics, the application of which has, inter alia, led to the discovery of the allotropic forms of hydrogen"
1933 Erwin Schrödinger and Paul Adrien Maurice Dirac "for the discovery of new productive forms of atomic theory"
1945 Wolfgang Pauli "for the discovery of the Exclusion Principle, also called the Pauli Principle"
© A simple model: standing wave of a confined string


A node (節點): zero amplitude
With one node:
$\lambda(=2 / 2 \lambda) \quad \lambda$ is smaller $\Rightarrow$ energy is higher

$$
\mathrm{E}=h v=h \mathrm{c} / \lambda
$$

With two nodes:
$\lambda$ is even smaller $\Rightarrow E$ is even higher

There are limitations: $l=n(1 / 2 \lambda)$ or $\lambda=2 l / n$
$n=1, \lambda=2 l$
$n=2, \lambda=l$

If in a circle


Limitations: $2 \pi r=n \lambda \quad n=1,2,3 \ldots \ldots$
Apply de Broglie equation:

$$
\begin{aligned}
\lambda=\frac{h}{m v} & \Rightarrow 2 \pi r=n \lambda=\frac{n h}{m v} \\
& \Rightarrow m v r=\frac{n h}{2 \pi}=n \hbar
\end{aligned}
$$


※ Heisenberg's uncertainty principle

In fact, the exact path of $\mathrm{e}^{-}$can not be determined

Uncertainty of

$$
\Delta x \cdot \Delta p \geqq \frac{h}{4 \pi}
$$

particle position
momentum
$\Delta(m v)$ : uncertainty of particle momentum

Ex. Hydrogen atom: r~0.05nm
Assume positional accuracy of $\mathrm{e}^{-}: 1 \%$ of $r$
Q: $\Delta v$ ?
Soln: $\Delta x=(0.05 \mathrm{~nm})(0.01)=5 \times 10^{-4} \mathrm{~nm}=5 \times 10^{-13} \mathrm{~m}$
$m_{\mathrm{e}}=9.11 \times 10^{-31} \mathrm{~kg}$
$h=6.626 \times 10^{-34} \mathrm{~kg} \cdot \mathrm{~m}^{2} / \mathrm{s}$
$\Delta x \cdot \Delta p=\frac{h}{4 \pi} \quad \Rightarrow \quad \Delta x \cdot m \Delta v=\frac{h}{4 \pi}$
$\Delta v=\frac{h}{4 \pi m \Delta x}=1.15 \times 10^{8} \mathrm{~m} / \mathrm{s} \leftarrow$ highly inaccurate
However, for a ball with $r=0.05 \mathrm{~m}, m=0.2 \mathrm{~kg}$
$\Delta x=(0.05 \mathrm{~m})(0.01)=5 \times 10^{-4} \mathrm{~m}$
$\Delta v=5 \times 10^{-31} \mathrm{~m} / \mathrm{s} \longleftarrow$ very accurate
$\Rightarrow$ Macroscopically, no problem
※ The particle in a box

A hypothetical situation
1．Illustrate the math
2．Show some characteristics of wave function
3．Show how E quantization occurs
The model

Particle mass：$m$


One dimensional movement（on x－axis）
The only possible energy is KE

In Schrödinger equation

$$
\begin{aligned}
& \text { The operator for KE: }-\frac{\hbar^{2}}{2 m} \frac{d^{2}}{d x^{2}} \\
& \hat{H} \Psi=E \Psi \quad \Rightarrow \quad-\frac{\hbar^{2}}{2 m} \frac{d^{2} \psi}{d x^{2}}=E \psi
\end{aligned}
$$

A function of $x$

The connection with classical physics
The classical wave equation for standing wave：

$$
\begin{aligned}
& y(x, t)=2 A \sin \left(\frac{2 \pi}{\lambda} x\right) \cos (2 \pi v t) \\
& \Rightarrow \quad \frac{d^{2} y(x)}{d x^{2}}=-\left(\frac{2 \pi}{\lambda}\right)^{2} y(x)
\end{aligned}
$$

Use de Broglie eq．$\frac{1}{\lambda}=\frac{p}{h} \quad \Rightarrow \quad \frac{d^{2} y(x)}{d x^{2}}=-\frac{p^{2}}{\hbar^{2}} y(x)$
$E=\frac{1}{2} m v^{2}=\frac{p^{2}}{2 m} \Rightarrow p^{2}=2 m E \Rightarrow \frac{d^{2} y(x)}{d x^{2}}=-\frac{2 m E}{\hbar^{2}} y(x)$

$$
\begin{aligned}
& -\frac{\hbar^{2}}{2 m} \frac{d^{2} \psi}{d x^{2}}=E \psi \\
\Rightarrow & \frac{d^{2} \psi}{d x^{2}}=-\frac{2 m E}{\hbar^{2}} \psi
\end{aligned}
$$

$2^{\text {nd }}$ derivative of a function = the same function
$\Rightarrow$ Points to a sine function
Ex. $\frac{d^{2}}{d x^{2}}(A \sin k x)=A \frac{d}{d x}\left(\frac{d \sin k x}{d x}\right)=A \frac{d}{d x}(k \cos k x)$

$$
\begin{aligned}
& =A k \frac{d \cos k x}{d x}=A k(-k \sin k x) \\
& =-k^{2} A \sin k x
\end{aligned}
$$

## Compare

$$
\frac{d^{2} \psi}{d x^{2}}=-\frac{2 m E}{\hbar^{2}} \psi \quad \text { and } \quad \frac{d^{2}}{d x^{2}}(A \sin k x)=-k^{2} A \sin k x
$$

If $A \sin k x=\Psi \Rightarrow \frac{d^{2} \psi}{d x^{2}}=-k^{2} \psi$
$\Rightarrow \quad k^{2}=\frac{2 m E}{\hbar^{2}} \quad \Rightarrow \quad E=\frac{\hbar^{2} k^{2}}{2 m}$
A, $k$ ?
Apply boundary conditions (to make sense):

1. Must be confined in the box
2. The total probability of finding the particle $=1$
3. The wave function must be continuous

$$
\Psi=A \sin (k x)
$$

1. At $x=0 \quad \Psi=0$

At $x=L \quad \Psi=0$
for $\sin \theta=0 \Rightarrow \theta=0^{\circ}, 180^{\circ}, 360^{\circ}, \ldots$.

$$
\begin{array}{ll}
\text { At } x=0 & \Psi=A \sin (k x)=A \sin (0)=0 \\
\text { At } x=L & \Psi=A \sin (k L)=0 \\
& \Rightarrow k L=n \pi \quad \Rightarrow \quad k=\frac{n \pi}{L} \quad n=1,2,3, . .
\end{array}
$$

2. Important: The physical meaning of $\Psi$ is that $\Psi^{2}$ is the probability

Thus: Total probability of finding the $\quad=\int_{0}^{L} \psi^{2}(x) d x=1$ particle in the box

$$
\begin{aligned}
& \Psi(x)=A \sin (k x) \\
\Rightarrow & \Psi^{2}(x)=A^{2} \sin ^{2}(k x)=A^{2} \sin ^{2}\left(\frac{n \pi}{L} x\right) \\
\Rightarrow & \int_{0}^{L} A^{2} \sin ^{2}\left(\frac{n \pi}{L} x\right) d x=1 \\
\Rightarrow & \underbrace{\int_{0}^{L} \sin ^{2}\left(\frac{n \pi}{L} x\right) d x}_{\text {II }}=\frac{1}{A^{2}} \quad \begin{array}{l}
\text { Ref: } \\
\int \sin ^{2}(a x) d x=\frac{1}{2} x-\frac{1}{4 a} \sin (2 a x)
\end{array} \\
\Rightarrow & \frac{L}{2}=\frac{1}{A^{2}} \Rightarrow A=\sqrt{\frac{2}{L}}
\end{aligned}
$$

$$
\begin{aligned}
& \Psi=A \sin (k x) \quad A=\sqrt{\frac{2}{L}} \quad k=\frac{n \pi}{L} \quad n=1,2,3, \ldots \\
& \Rightarrow \quad \psi(x)=\sqrt{\frac{2}{L}} \sin \left(\frac{n \pi}{L} x\right) \\
& E=\frac{\hbar^{2} k^{2}}{2 m}=\frac{\hbar^{2}(n \pi / L)^{2}}{2 m}=\frac{n^{2} h^{2}}{8 m L^{2}} \\
& n=1 \quad \psi_{1}=\sqrt{\frac{2}{L}} \sin \left(\frac{\pi}{L} x\right) \quad E_{1}=\frac{n^{2}}{8 m L^{2}} \\
& n=2 \quad \psi_{2}=\sqrt{\frac{2}{L}} \sin \left(\frac{2 \pi}{L} x\right) \quad E_{2}=\frac{4 h^{2}}{8 m L^{2}}=\frac{h^{2}}{2 m L^{2}}
\end{aligned}
$$



Boundary cond. $\Rightarrow$ quantized E level, $n$ : quantum \# $n=0 \Rightarrow \Psi(x)=0, \Psi^{2}(\mathrm{x})=0 \Rightarrow$ can't be true
$\Rightarrow n \neq 0$, lowest $E \neq 0$
$\Rightarrow n=1$ having the lowest $E$ (zero-point energy)

## 曹灣大楽化䵓系 <br> NTU ©hemistry

※ The wave equation for the hydrogen atom
$\checkmark$ The electron movement in three dimension is considered
$\checkmark$ Potential E due to charge－charge attraction is included
$\Rightarrow$ Apply boundary conditions
$\Rightarrow$ Solve the differential equation
$\Rightarrow$ Obtain a set of solutions： the wave functions for an electron

The quantum numbers appear：
$n$－the principal quantum number
I－the angular momentum quantum number
$m_{l}-$ the magnetic quantum number

Using spherical polar coordinate

$$
\psi_{n, l, m}(r, \theta, \phi)=R_{n, l}(r) \Theta_{l, m}(\theta) \Phi_{m}(\phi)
$$

$n=1, l=0, m_{l}=0 \quad$ 1s orbital
$\psi_{\text {is }}=\frac{1}{\sqrt{\pi}}\left(\frac{z}{a_{0}}\right)^{3 / 2} e^{-\sigma} \quad Z=1$ for hydrogen $\quad \sigma=\frac{Z r}{a_{0}}$

$$
\begin{aligned}
& a_{o}=\frac{\varepsilon_{0} h^{2}}{\pi m e^{2}}=5.29 \times 10^{-11} \mathrm{~m} \\
& \left(\varepsilon_{0}=8.85 \times 10^{-12} \mathrm{C}^{2} \mathrm{~J}^{-1} \mathrm{~m}^{-1}\right)
\end{aligned}
$$

$\uparrow$ Vacuum permittivity（真空介電係數）
$n=2, l=1, m_{l}=+1 \quad 2 p_{x}$ orbital
$\psi_{2 p_{x}}=\frac{1}{4 \sqrt{2 \pi}}\left(\frac{z}{a_{0}}\right)^{3 / 2} \sigma e^{-\sigma / 2} \sin \theta \cos \phi$
$\checkmark$ Energies of hydrogen＇s electron

$$
\begin{aligned}
E_{n} & =-\frac{z^{2}}{n^{2}}\left(\frac{m e^{4}}{8 \varepsilon_{0}^{2} h^{2}}\right)=-2.178 \times 10^{-18} \mathrm{~J}\left(\frac{z^{2}}{n^{2}}\right) \\
\uparrow & \uparrow=1,2,3, . .
\end{aligned}
$$

$E$ depends only on $n$
（true for one electron system）
Same as derived from Bohr＇s model
Cf．Bohr＇s model

$$
E=-\frac{2 \pi^{2} m e^{4}}{h^{2}}\left(\frac{z^{2}}{n^{2}}\right)=-2.178 \times 10^{-18} \mathrm{~J}\left(\frac{Z^{2}}{n^{2}}\right)
$$

（expressed in cgs unit）


Wave function $\Psi$ ：
Describes the state of a system
Contains information about all the properties of the system that are open to experimental determination By uncertainty principle，it is difficult to know the exact position and direction of movementBorn interpretation：

$$
\frac{\Psi_{1}{ }^{2}}{\Psi_{2}{ }^{2}}=\frac{N_{1}}{N_{2}} \text { Probability of finding } \mathrm{e}^{-} \text {at position } 1
$$

$\Psi^{2}$ ：a function about probability distribution
Postulate：
The probability that a particle will be found in the volume element $d \tau$ at the point $r$ is proportional to $|\Psi(\mathrm{r})|^{2} d \tau$

Ex. 1s orbital for H atom


The real probability distribution:

$$
\Psi^{2} \cdot\left(4 \pi r^{2}\right)
$$

The most probable distance to find $\mathrm{e}^{-}$
$\Rightarrow$ Same as based on
Bohr model ( $n=1$ )
$\Rightarrow$ Called Bohr radius
Summary

Bohr model: a fixed path

Quantum mechanics: a probability

Normally the pictorial boundary shows $90 \%$ probability inside the boundary
(for 1 s of $\mathrm{H}, r=1.4 \AA$ )
Note:
Simple pictorial models
always oversimplify the phenomenon

$\checkmark$ The angular quantum number: $l$ (integer)
For each $n, l=0-n-1$
Related to the angular momentum of an $\mathrm{e}^{-}$ Determines the shape

$$
\begin{array}{ll}
l=0 & \text { s orbital } \\
l=1 & \text { p orbital } \\
l=2 & \text { d orbital } \\
l=3 & \text { f orbital }
\end{array}
$$

$n=1 \quad l=0 \Rightarrow 1 s$
$n=2 \quad l=0 \Rightarrow 2 \mathrm{~s}$
$n=2 \quad l=1 \Rightarrow 2 p$
$\checkmark$ The magnetic quantum number: $m_{l}$ (integer)

$$
\left.m_{l}=l \ldots \ldots-l \text { (including } 0\right)
$$

Related to the orientation in space

$$
\begin{array}{ll}
l=1 \Rightarrow m_{l}=1,0,-1 & \Rightarrow \mathrm{p}_{\mathrm{x}}, \mathrm{p}_{\mathrm{y}}, \mathrm{p}_{\mathrm{z}} \\
l=2 \Rightarrow m_{l}=2,1,0,-1,-2 \Rightarrow \mathrm{~d}_{\mathrm{z}^{2}}, \mathrm{~d}_{\mathrm{x}^{2}-\mathrm{y}^{2}}, \mathrm{~d}_{\mathrm{xy}}, \mathrm{~d}_{\mathrm{yz}}, \mathrm{~d}_{\mathrm{zx}}
\end{array}
$$

© Summary
$n$ determines the total $\mathrm{E}: \quad E_{n}=-\frac{1}{n^{2}}\left(\frac{Z^{2} e^{2}}{2 a_{o}}\right)$
$l$ determines the square of the total angular momentum:

$$
M^{2}=l(l+1) \hbar^{2}
$$

$m_{l}$ determines the $z$ component of the angular momentum:

$$
M_{z}=m \hbar
$$


$\checkmark$ Probability distribution of $p$ orbitals


Contain two lobes$\psi_{2 p_{z}}=\frac{1}{4 \sqrt{2 \pi}}\left(\frac{z}{a_{0}}\right)^{3 / 2} \sigma e^{-\sigma / 2} \cos \theta$

Three subshells




## $\checkmark$ d subshells: 5 orbitals

$$
\begin{array}{lll}
m_{l}=0 & \pm 1 & \pm 2 \\
\mathrm{~d}_{\mathrm{z}^{2}} & \mathrm{~d}_{\mathrm{xz}} & \mathrm{~d}_{\mathrm{xy}} \\
& \mathrm{~d}_{\mathrm{yz}} & \mathrm{~d}_{\mathrm{x}^{2}-\mathrm{y}^{2}}
\end{array}
$$


$d_{x y}$

$d_{x z}$

$d_{y z}$

$d_{x^{2}-y^{2}}$

$\checkmark$ The energy level
For H atom: E is determined by $n$
same $n \Rightarrow$ same E
$\Rightarrow$ these orbitals are degenerate

※ Electron spin and Pauli principle

##  <br> NTU © hemistry

$\checkmark$ In fact, a spin quantum number $\left(m_{\mathrm{s}}\right)$ exists

$$
m_{\mathrm{s}}=+1 / 2 \text { or }-1 / 2
$$

Electron has its own angular momentum
$\Rightarrow$ Imagine the electron as spinning on its own axis like earth
$\Rightarrow$ Behaves like a tiny magnet
$\checkmark$ Pauli (1900 - 1958) principle
In a given atom, two electrons can not have the same $n, l, m_{l}$ and $m_{\mathrm{s}}$
$\Rightarrow \quad$ In the same orbital, $n, l$, and $m_{l}$ must be the same
$\Rightarrow \quad m_{\mathrm{s}}$ must be different
※ Polyelectronic atoms

NTU chemistry

Very complicate
Problem: Electrons influence each other
Ex. $\mathrm{He}+2 \mathrm{e}^{-}$
$\downarrow-\mathrm{e}^{-} \quad 2372 \mathrm{~kJ}$
$\mathrm{He}^{+}$
© The effective charge approximation
(A very rough model)
Considering $\mathrm{e}^{-}-\mathrm{e}^{-}$repulsion as reducing the nuclear charge
In other words:
The electron is shielded (screened) from the nuclear charge by the other electrons
Ex. Hypothetical He


Becomes a one $\mathrm{e}^{-}$system
$\Rightarrow$ Leads to hydrogen like orbitals
$\Rightarrow$ But the sizes and energies are different from that of H atom

Gives only a qualitative picture
(similar to the hydrogen model)
Quantitatively not accurate
※ More about polyelectronic model

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In hydrogen
Schrödinger equation can be solved exactly
In polyelectronic atoms
Schrödinger equation can not be solved exactly
Approximations are required
The self-consistent field (SCF) method can be used
$\checkmark$ The self-consistent field (SCF) method
Considering the electron as residing in a field
Composed of the nucleus
and other electrons (in their various orbitals)
The differential equation contains four parts

1. The usual KE contribution
2. $V_{\text {electron-nucleus }}$ (attraction)
3. The PE due to the charge density of the $\mathrm{e}^{-} \mathrm{s}$ in the other occupied orbitals
4. The spin correlation effect
$\Rightarrow$ A set of one electron equations can be obtained and solved using successive approximation


Most probable distance is smaller for $2 p$
But $E_{2 s}<E_{2 p}$ because of the penetrating effect of 2s
Similarly, $\mathrm{E}_{4 \mathrm{~s}}<\mathrm{E}_{3 \mathrm{~d}}$
More penetrating

※ The aufbau principle（遞建原理） and the periodic table

1869 Mendeleev
The first periodic table
A correlation of chemical properties and AW of elements
© The periodicity based on quantum mechanics
The aufbau principle：
As the atomic number increased the electrons are added in order


> Hund's rule: the lowest-energy config. Is the one having maximum number of unpaired $\mathrm{e}^{-} \mathrm{s}$ in a set of degenerate orbitals


Elements with the same valence electronic configuration
$\Rightarrow$ Show similar chemical behavior
$\Rightarrow$ Grouped in the vertical column

$$
\begin{array}{ll}
\mathrm{Li} & {[\mathrm{He}] 2 \mathrm{~s}^{1}} \\
\mathrm{Na} & {[\mathrm{Ne}] 3 \mathrm{~s}^{1}} \\
\mathrm{~K} & {[\mathrm{Ar}] 4 \mathrm{~s}^{1}}
\end{array}
$$

Some notes

1. $(n+1) s$ before $n d$
2. After lanthanide (La: $[\mathrm{Xe}] 6 \mathrm{~s}^{2} 5 \mathrm{~d}^{1}$ )
$\Rightarrow$ starts to fill in 4f
Ce: $[X e] 6 s^{2} 4 f^{1} 5 d^{1}$
$\Rightarrow$ the lanthanide series
3. After actinide (Ac: $\left.[R n] 7 s^{2} 6 d^{1}\right)$
$\Rightarrow$ fill in $5 f$
$\Rightarrow$ actinide series

lanthanides:
actinides:

4. Group labels: 1A, 2A 8A

The total number of valance $\mathrm{e}^{-}$
5. $1 \mathrm{~A}-8 \mathrm{~A}$ : the main group elements
※ Periodic trends in atomic properties
© Ionization energy

$$
\begin{aligned}
& \quad{ }^{\mathrm{X}}(\mathrm{~g}) \rightarrow \mathrm{X}^{+}(g)+\mathrm{e}^{-} \quad \text { energy change } \\
& \text { Atom } \\
& \text { or ion Gas phase }
\end{aligned}
$$

Sometimes expressed as ionization potential
unit: eV $=1.602 \times 10^{-19} \mathrm{~J}$
$\sim 23 \mathrm{kcal} / \mathrm{mol}$
$\sim 96 \mathrm{~kJ} / \mathrm{mol}$

Ex. AI: [Ne]3s²3p ${ }^{1}$

$$
\begin{array}{ll}
\mathrm{Al}(g) \rightarrow \mathrm{Al}^{+}(g)+\mathrm{e}^{-} & \mathrm{I}_{1}=580 \mathrm{~kJ} / \mathrm{mol} \\
& \mathrm{I}_{2}=1815 \\
& \mathrm{I}_{3}=2740 \\
& \mathrm{I}_{4}=11600
\end{array}
$$

$\mathrm{I}_{1}$ : The first ionization E
$\Rightarrow$ Removes the highest-E e ${ }^{-}$
$\Rightarrow$ Reflect the E of the orbital
$\mathrm{I}_{2}$ : The second ionization E
$\Rightarrow$ The charge effect comes to play
$\mathrm{I}_{4}$ : very large ( $\left.\mathrm{Al}^{3+}:[\mathrm{Ne}]\right)$
$\Rightarrow$ Starts to remove core $\mathrm{e}^{-}$

$$
\left[\begin{array}{llllllll}
\mathrm{Ne}] 3 s^{1} & 3 s^{2} & 3 s^{2} 3 p^{1} & 3 s^{2} 3 p^{2} & 3 s^{2} 3 p^{3} & 3 s^{2} 3 p^{4} & 3 s^{2} 3 p^{5} & 3 s^{2} 3 p^{6}
\end{array}\right.
$$

|  | Na | Mg | Al | Si | P | S | Cl | Ar |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{I}_{1}$ | 495 | 735 | 580 | 780 | 1060 | 1005 | 1255 | 1527 |
| $(\mathrm{~kJ} / \mathrm{mol})$ |  |  |  |  |  |  |  |  |

General trend $\rightarrow$ increasing

Shielding effect of core $\mathrm{e}^{-} \Rightarrow$ similar Increasing of $Z^{+}$eff $\Rightarrow$ more important

Special case
Al: the lower value is due to the shielding effect of $3 s^{2}$
S : the lower value is due to pairing energy

$$
\text { ( } \mathrm{e}^{-}-\mathrm{e}^{-} \text {repulsion) }
$$

Down a group

|  | $\mathrm{I}_{1}(\mathrm{~kJ} / \mathrm{mol})$ | size |  |
| :--- | :--- | :--- | :--- |
| Li | 520 |  |  |
| Na | 495 |  |  |
| K | 419 | decreasing |  |
| Rb | 409 |  | increasing |
| Cs | 382 |  |  |
|  |  |  |  |

$Z^{+}{ }_{\text {eff }}$ similar $\Rightarrow$ Size is more important
(0) Electron affinity

$$
\begin{array}{ll}
\mathrm{X}(g)+\mathrm{e}^{-} \rightarrow \mathrm{X}^{-}(g) & \begin{array}{l}
\text { energy change } \\
\Delta \mathrm{H}(-): \text { exothermic }
\end{array}
\end{array}
$$

In a period: atomic number $\uparrow$
energy change: more negative
Ex. C N
-122.5 not available $-141.4(\mathrm{~kJ} / \mathrm{mol})$

$\mathrm{Z}^{+}{ }_{\text {eff }} \longrightarrow$ increase
$\mathrm{N}^{-}$: unstable due to $\mathrm{e}^{-}-\mathrm{e}^{-}$repulsion

Down a group

|  | $\mathrm{kJ} / \mathrm{mol}$ |  |
| :--- | :--- | :--- |
| F | -327.8 | F is too small: |
| Cl | -348.7 | $\mathrm{e}^{-}-\mathrm{e}^{-}$repulsion is important |
| Br | -324.5 |  |
| I | -295.2 |  |
|  |  |  |
|  | Less negative |  |

$Z^{+}$eff similar $\Rightarrow$ Size is more important (the difference is not large)Atomic radius
difficult to determine just like orbitals
Usual way


For metals: treat similarly General trend

Across the period - size $\downarrow$

$Z^{+}$eff is more important
Down a group - size $\uparrow$
due to the increase of $n$

The alkali metals
$\mathrm{Li}, \mathrm{Na}, \mathrm{K}, \mathrm{Rb}, \mathrm{Cs}, \mathrm{Fr}$
Most reactive but rare in nature
d
mp
bp
IP
r $\qquad$
Good reducing agents

$$
\begin{aligned}
& 2 \mathrm{Na}(s)+\mathrm{Cl}_{2}(g) \rightarrow 2 \mathrm{NaCl}(s) \\
& 6 \mathrm{Li}(s)+\mathrm{N}_{2}(g) \rightarrow 2 \mathrm{Li}_{3} \mathrm{~N}(s)
\end{aligned}
$$

In general:
reducing ability
$\mathrm{Li}<\mathrm{Na}<\mathrm{K}<\mathrm{Rb}<\mathrm{Cs}$
In aqueous solution
$\mathrm{Li}>\mathrm{K}>\mathrm{Na}$
Ex. $\quad 2 \mathrm{M}(\mathrm{s})+2 \mathrm{H}_{2} \mathrm{O}(I) \rightarrow \mathrm{H}_{2}(g)+2 \mathrm{M}^{+}(a q)+2 \mathrm{OH}^{-}(a q)$

+ energy
Reason: $\mathrm{Li}^{+}$has high hydrating energy
$\mathrm{Li}^{+} \quad-500 \mathrm{~kJ} / \mathrm{mol} \longleftarrow$ Small size
$\mathrm{Na}^{+}-400 \quad$ high charge density
$\mathrm{K}^{+} \quad-300 \quad$ stronger interaction with $\mathrm{H}_{2} \mathrm{O}$

