# 實驗經濟學一：行為賽局論
Experimental Economics I: Behavioral Game Theory

## 第十四講：市場設計：臺灣國中會考
Lecture 14: Market Design at Taiwan

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本課程指定教材: Colin E. Camerer, *Behavioral Game Theory: Experiments in Strategic Interaction*. New York: Russell Sage Foundation; New Jersey: Princeton UP, 2003.

# 志願難填 教團：學生陷賽局困境

(2014/6/9 國語日報）國教行動聯盟昨天痛批，升學制度儼然變成賭博式賽局，學生想進理想學校，竟得猜測別人的志願怎麼填，陷入「賽局理論」困境。

- 國教行動聯盟理事長王立昇表示，志願序納入超額比序計分，填錯會被扣分，加上第一次免試分發後，基北區約有六千個學生可能放棄錄取考特招，所以預測別人填哪些志願、會不會放棄一免，成了填寫志願的重要因素。
- 王立昇指出，「賽局理論」是研究遊戲中個體預測對方和己方行為，所產生的影響，並分析最佳策略。現在的十二年國教，已經讓學生面臨一樣的困擾。

# 填志願諜對諜　國教盟驚爆：學生想輕生

國中會考成績上周四公布後，家長學生茫然不知如何選填志願。

國教行動聯盟今上午公開呼籲教育部，今年取消志願序計分，或採 3-7 個志願為群組，差一個群組扣 1 分，以免學生陷入選填志願的博弈賽局中，填志願淪為諜對諜。

# 填志願諜對諜　國教盟驚爆：學生想輕生

(2014/6/7 蘋果日報 )

國教行動聯盟理事長王立昇表示，…教育部應公布更多資訊並延長志願表繳交時間，讓學生有更充足資訊能錄取最理想的學校。他進一步表示，學生為了上好學校，同學間已互相猜忌，打探彼此第一志願是什麼做為自己選填志願的參考，陷入博弈賽局中，解決方法只有取消志願序計分，或擴大為群組計分，降低傷害。

# 制度變數多 教團憂入學如賽局（2014/6/8)

- （中央社記者許秩維）國教行動聯盟今天說，國教入學制度變數多，恐陷賽局理論，孩子得預測他人如何填志願，聯盟籲取消志願序計分。
- 國教行動聯盟舉行記者會，憂心國教入學制度陷入賽局理論的困境，讓學生和家長寢食難安。
- 國教行動聯盟理事長王立昇表示，目前國教入學制度面臨幾個問題，如志願序計分，由於不知別人如何填志願，要進入自己理想的學校就可能有很多變數，導致陷入賽局理論的困境，學生家長難以填志願。

# Taiwan High School Choice

- History School Choice in Taiwan
  - Old System: Gale-Shapley Deferred Acceptance
  - New System in 2014
- Exam-exempt School Choice based on:
  - # of ABC from Joint Exam ( 會考 )
  - Self-reported School Choice Rankings
  - Other factors (that all get the same score)
  - Chinese composition: Grade 1-6
  - A++, A+, A, A-, etc.

# Taiwan School Choice: A Simplified Model

- How can we analyze this?
  - Simplify to obtain a tractable model/example
  - Implement in the lab
- What are key elements of the situation?
- What are the key results to reproduce?
- Next: Run lab experiments to

1. Test the model
2. Try alternative institutions
3. Teach parents/policy makers

# Taiwan School Choice: A Simplified Model

- Three schools: *A, B, C*
- Three students: 1 & 2 are type *a*, 3 is type *c*
- Student Payoffs: $u(A) = h, \quad u(B) = 1, \quad u(C) = 0$
- School Payoffs: $v(a) = 1, \quad v(c) = 0$
- Actions: Self-report School Choice Rankings

$$S = \{ABC, BAC, ACB, CAB, CBA, BCA\}$$

- Assign everyone to their first choice
  - Ties broken by student type (grade), then random
  - Remaining students assigned to remaining schools

# Taiwan School Choice: A Simplified Model

- This is manipulable (=not strategy-proof)
  - Truthful Reporting of Ranking is not BR!
- Suppose all students truthfully report *ABC*
- Outcome: Student 1, 2 go to schools *A, B* (randomly) and student 3 goes to school *C*
  - Schools *ABC* get students of type *aac*

$$U_3(\underline{BAC}) = u(B) = 1 > u(C) = 0 = U_3(ABC)$$

- But: Student 3 could gain by misreporting!

# Taiwan School Choice: A Simplified Model

- What is the Nash Equilibrium of the game?

1. Student 3 reports *BAC*
2. Student 1 & 2 report *ABC* with prob. $p$, report *BAC* with prob. $(1 - p)$

- Outcome:
- $p^2$ : School *ABC* get students of type *aca*
  - When both Student 1 & 2 report ABC…
- $1 - p^2$: School *ABC* get students of type *aac*

# Why is this a Nash Equilibrium?

3 reports *BAC*; 1,2 report *ABC*/ *BAC* with ($p$,$1 - p$)

- For Student 1 (and 2) to mix, need: $\boxed{1 + p = h}$

$$U_1(ABC) = p\left(\frac{1}{2} \cdot u(A) + \frac{1}{2} \cdot u(C)\right) + (1-p) \cdot u(A)$$

$$= p\left(\frac{1}{2} \cdot h + \frac{1}{2} \cdot 0\right) + (1-p) \cdot h \quad = \left(1 - \frac{p}{2}\right) h$$

$$U_1(BAC) = p \cdot u(B) + (1-p)\left(\frac{1}{2} \cdot u(B) + \frac{1}{2} \cdot u(A)\right)$$

$$= p \cdot 1 + (1-p)\left(\frac{1}{2} \cdot 1 + \frac{1}{2} \cdot h\right) \quad = \frac{1+p}{2} + \frac{1-p}{2} \cdot h$$

# Taiwan School Choice: A Simplified Model

- Why is this a Nash Equilibrium?
  - Student 1 & 2 report *ABC* with prob. $\boxed{p = h - 1}$
- For Student 3, we need $\boxed{p > 0.555(0.55496)}$

$$
\begin{aligned}
f(p) &= U_3(BAC) - U_3(ABC) \geq 0 \\
&= p^2 \cdot 1 - (1-p)^2 \cdot h \\
&= p^2 - (1-p) \cdot (1-p^2)
\end{aligned}
$$

Since $f(p)$ increasing

$$f'(p) = 2p + (1-p^2) + 2p(1-p) > 0$$

$$\Rightarrow 1 + p = h > 1.555(0.55496)$$

# Conclusion (for the Example) 結論

- Nash Equilibrium of this 3-student game:

1. Student 3 untruthfully reports *BAC*
2. Student 1 & 2 mix between truthful & untruthful reports *ABC*/ *BCA*, ($p$, $1-p$)

- Outcome:
- $p^2$ : School *ABC* get students of type *aca*
  - When both Student 1 & 2 report ABC…
- $1-p^2$: School *ABC* get students of type *aac*

# Possible Extensions:

1. Is Cardinal Utility Required?
   - Ordinal preferences is fine if exists $p$ so that

$$\left(\frac{p}{2}\right) \cdot C + \left(1 - \frac{p}{2}\right) \cdot A \sim \left(\frac{1+p}{2}\right) \cdot B + \left(\frac{1-p}{2}\right) \cdot A$$

2. What if students have different preferences?
   - Different Risk Attitudes?
3. What if there are more students/schools?
4. What if schools can also act strategically?
5. What is a Good Alternative Mechanism?

# A Simple Theory of Matching (R-S, Ch.2)

- Gale & Shapley (1962); Roth & Sotomayor (1990)

- Finite Set of Students $S$ and Schools $C$
- 1-1 Matching, Strict (Ordinal) Preferences:
  - $c \succ_s \tilde{c}$ : Student $s$ prefers School $c$ to $\tilde{c}$
  - $s \succ_c \tilde{s}$ : School $c$ prefers Student $s$ to $\tilde{s}$
  - $i \succ_j \emptyset$ : $i$ is acceptable to $j$
- A matching is $\mu : S \cup C \to S \cup C \cup \{\emptyset\}$

$$\underbrace{\mu(s)}_{\in C \cup \{\emptyset\}} = c \quad \Leftrightarrow \quad \underbrace{\mu(c)}_{\in S \cup \{\emptyset\}} = s$$

# A Simple Theory of Matching (R-S, Ch.2)

- Matching $\mu$ blocked by individual $i$ if $\emptyset \succ_i \mu(i)$
- Matching $\mu$ blocked by pair *s*, *c* if
  - $c \succ_s \mu(s)$ and $s \succ_c \mu(c)$
- Matching is stable if it is blocked by neither
  - Core = Set of all stable matchings
  - A stable matching is Pareto efficient
- Theorem (Gale-Shapley, R-S Theorem 2.8)
  - Exists a stable matching in any 1-1 matching market

# Deferred Acceptance Algorithm

- Step 1: Students apply to their first choices
  - Schools tentatively hold most preferred student and reject all others
- Step $t$ (2 and above): Students rejected in Step $t - 1$ apply to next highest choice
  - Schools tentatively hold most preferred student (new or held) and reject all others
- Stop when no more new applications
  - Happens in finite time!

# DA Algorithm: Taiwan School Choice Model

- 3 schools: *A, B, C*; 3 students: *a, b, c*
  - Student Payoffs: $u(A) = h, \quad u(B) = 1, \quad u(C) = 0$
  - School Payoffs: $v(a) = 1, \quad v(b) = 0.999, \quad v(c) = 0$
- Step 1: All students apply to school *A*
  - School *A* holds student *a* and rejects *b, c*
- Step 2: Students *b, c* apply to school *B*
  - School *B* holds student *b* and rejects *c*
- Step 3: Students *c* applies to school *C*
  - School *C* holds student *c* and terminates DA!

# Deferred Acceptance Algorithm

- Proof of Theorem (Gale-Shapley)
  - DA gives matching where no student/school applies to/holds unacceptable schools/students

➢ Matching $\mu$ not blocked by any individual!
  - If $c \succ_s \mu(s) \neq c$, $s$ was rejected by $c$ before in DA
  - But in DA, $c$ rejects only if it sees better choice!
  - Hence, $\mu(c) \succ_c s$

➢ Matching $\mu$ not blocked by any pair!

- Resulting Matching $\mu$ of DA is stable. QED

# DA Algorithm: Taiwan School Choice Model

- What does stable mean in the field?!
- Roth (1984):
  - stable ones successfully used
  - continue to be used (unstable ones abandoned)
- Few complaints in Taiwan?!
- A student-proposing DA algorithm yields:
- Student-optimal stable matching
  - (superior to all other stable matching)
  - Proof of Theorem? See R-S Theorem 2.12

# DA Algorithm: Marriage Matching

- Male-optimal stable matching
  - (superior to all other stable matching)

= Female-pessimal
  - (inferior to all other stable matching)

- In contrast, A female-proposing DA leads to
  - Female-optimal/male-pessimal stable matching
- Why is proposing power less important school choice?
  - Student/School Preferences More Aligned?

# Rural Hospital Theorem (R-S Theorem 2.22)

- The same set of students/schools are left unmatched in all stable matching
- This means:
  - A loser is a loser in any stable matching ( 魯蛇到哪裡都是魯蛇 )
  - Cannot expect any stable-matching mechanism to solve rural hospital problem ( 偏遠地區醫療 )
- Proof?

# Proof of Rural Hospital Theorem

- Student-optimal stable matching $\overline{\mu}$
- Alternative stable matching $\mu$
- $\overline{\mu}$ is student-optimal:
  - Students matched in $\mu$ also matched in $\overline{\mu}$
- $\overline{\mu}$ is school-pessimal:
  - Schools matched in $\overline{\mu}$ also matched $\mu$
- # of matches are the same in any match
- Same set of students/schools matched in $\overline{\mu}, \mu$

# Truthful Reporting and Strategy-Proofness

- Main problem of the new system in Taiwan:
  - People want to misrepresent their preferences!
- Mechanism: Rule that yields a matching from (reported) preferences
- A mechanism is strategy-proof if reporting true preferences is a dominant strategy for everyone
  - The new system in Taiwan is not strategy-proof
  - Is DA strategy-proof?

# Truthful Reporting and Strategy-Proofness

- In fact, no stable mechanism is strategy-proof! (R-S Theorem 4.4)
  - But, by Dubins and Freedman 1981, Roth 1982:
- Theorem (R-S Theorem 4.7): The student-proposing DA is strategy-proof for students.
- Why DA (old system in Taiwan) is good:
  1. Stable
  2. Students prefer it to all other stable matching
  3. Strategy-proof for students

# Extensions:

1. Strategy-proof ☒ Manipulable
   - Degree of strategy-proofness (instead of Y/N)
2. 1-1 ☒ Many-to-one
   - Schools can accept up to $q_c$ students (quota)
   - Existence of stable many-to-one matching market
   - X-proposing DA ☒ X-optimal stable matching
   - Rural Hospital Theorem (fill same # of students)
   - Student-proposing DA strategy-proof for students
   - No stable mechanism strategy-proof for schools
3. Problem for Married Couples?!

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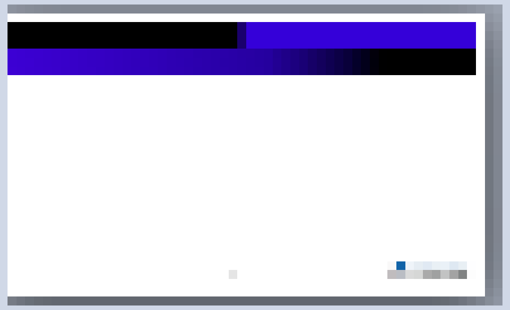
| 1-26 |  | BY NC SA | 國立臺灣大學 經濟學系 王道一 教授 |
| 2 | 國教行動聯盟昨天痛批，升學制度儼然…現在的十二年國教，已經讓學生面臨一樣的困擾。 | | 劉偉瑩，國語日報，〈志願難教 教團：學生陷賽局困境〉，2014/6/9，引用自臺北市教育 e 週報 http://enews.tp.edu.tw/paper_show.aspx?EDM=EPS20140610144609J3Q 瀏覽日期：2016/2/20，依據著作權法第 46、52、65 條合理使用 |
| 3-4 | 國中會考成績上周四公布後…以免學生陷入選填志願的博弈賽局中…解決方法只有取消志願序計分，或擴大為群組計分，降低傷害。 | | 陳威廷，蘋果日報，〈填志願諜對諜 國教盟驚爆：學生想輕生〉，2014/06/08 http://www.appledaily.com.tw/realtimenews/article/new/20140608/412340/ 瀏覽日期：2016/2/20，依據著作權法第 46、52、65 條合理使用 |

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| 5 | 國教行動聯盟今天說，國教入學制度變數多，恐陷賽局理論…．要進入自己理想的學校就可能有很多變數，導致陷入賽局理論的困境，學生家長難以填志願。 |  | 許秩維，中央社，〈制度變數多 教團憂入學如賽局〉，引用自中時電子報， 2014/06/8<br>http://www.chinatimes.com/realtimenews/20140608002093-260405<br>瀏覽日期： 2016/2/20<br>依據著作權法第 46 、 52 、 65 條合理使用 |
| 8-9 | $u(A)=h,\quad u(B)=1,\quad u(C)=0$<br>$v(a)=1,\quad v(c)=0$<br>$S=\{ABC,BAC,ACB,CAB,CBA,BCA\}$<br>$U_3(BAC)=u(B)=1>u(C)=0=U_3(ABC)$<br>$\boxed{1+p=h}$ |  | 國立臺灣大學 經濟學系 王道一 教授<br>如需引用，請參考盧士彧，〈臺灣高中入學分析〉，臺灣大學社會科學院經濟學系碩士論文， pp.25-27.<br>依據著作權法第 46 、 52 、 65 條合理使用 |
| 11 | $U_1(ABC)=p\left(\frac{1}{2}\cdot u(A)+\frac{1}{2}\cdot u(C)\right)+(1-p)\cdot u(A)$<br>$=p\left(\frac{1}{2}\cdot h+\frac{1}{2}\cdot 0\right)+(1-p)\cdot h$<br>$U_1(BAC)=p\cdot u(B)+(1-p)\left(\frac{1}{2}\cdot u(B)+\frac{1}{2}\cdot u(A)\right)$<br>$=p\cdot 1+(1-p)\left(\frac{1}{2}\cdot 1+\frac{1}{2}\cdot h\right)$<br>$=\frac{1+p}{2}+\frac{1-p}{2}\cdot h$<br>$=p^2\cdot 1-(1-p)^2\cdot h$<br>$\boxed{p=h-1}$<br>$\boxed{p>0.555(0.55496)}$ |  | 國立臺灣大學 經濟學系 王道一 教授<br>如需引用，請參考盧士彧，〈臺灣高中入學分析〉，臺灣大學社會科學院經濟學系碩士論文， pp.25-27.<br>依據著作權法第 46 、 52 、 65 條合理使用 |
| 12 | $=p^2-(1-p)\cdot(1-p^2)$<br>$f(p)=U_3(BAC)-U_3(ABC)\geq 0$<br>$f'(p)=2p+(1-p^2)+2p(1-p)>0$<br>$\Rightarrow 1+p=h>1.555(0.55496)$ |  | 國立臺灣大學 經濟學系 王道一 教授<br>如需引用，請參考盧士彧，〈臺灣高中入學分析〉，臺灣大學社會科學院經濟學系碩士論文， pp.25-27. |

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| 14 | $\left(\frac{p}{2}\right)\cdot C+\left(1-\frac{p}{2}\right)\cdot A\sim$ $\left(\frac{1+p}{2}\right)\cdot B+\left(\frac{1-p}{2}\right)\cdot A$ | | 國立臺灣大學 經濟學系 王道一 教授<br>如需引用，請參考盧士彧，〈臺灣高中入學分析〉，臺灣大學社會科學院經濟學系碩士論文， pp.25-27.<br>依據著作權法第 46 、 52 、 65 條合理使用 |
| 15 | $\mu : S\cup C\to S\cup C\cup\{\emptyset\}$ $\underset{\in C\cup\{\emptyset\}}{\mu(s)}=c \quad\Leftrightarrow\quad \underset{\in S\cup\{\emptyset\}}{\mu(c)}=s$ | | A.E. Roth and Marilda Sotomayer, Two-Sided Matching, Cambridge UP, 1992, pp.17-20.<br>依據著作權法第 46 、 52 、 65 條合理使用 |
| 16 | Theorem (Gale-Shapley, R-S Theorem 2.8)<br>-Exists a stable matching in any 1-1 matching market | | A.E. Roth and Marilda Sotomayer, Two-Sided Matching, Cambridge UP, 1992, pp.27.<br>依據著作權法第 46 、 52 、 65 條合理使用 |
| 17 | Step 1: Students apply to their first choices… new applications. Happens in finite time! | | A.E. Roth and Marilda Sotomayer, Two-Sided Matching, Cambridge UP, 1992, pp.28-30.<br>依據著作權法第 46 、 52 、 65 條合理使用 |

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| 18 | $u(A) = h,\ \ u(B) = 1,\ \ u(C) = 0$<br>$v(a) = 1,\ \ v(b) = 0.999,\ \ v(c) = 0$ | | 國立臺灣大學 經濟學系 王道一 教授<br>如需引用，請參考盧士彧，〈臺灣高中入學分析〉，臺灣大學社會科學院經濟學系碩士論文， pp.25-27.<br>依據著作權法第 46 、 52 、 65 條合理使用 |
| 19 | Proof of Theorem (Gale-Shapley) DA gives matching...Resulting Matching of DA is stable. QED | | A.E. Roth and Marilda Sotomayer, Two-Sided Matching, Cambridge UP, 1992, pp.30-32.<br>依據著作權法第 46 、 52 、 65 條合理使用 |
| 20 | stable ones successfully used continue to be used (unstable ones abandoned) | | A.E. Roth, “The Evolution of the Labor Market for Medical Interns and Residents: A Case Study in Game Theory,” Journal of Political Economy, Vol.92, No.6, (1984), pp. 995-996 and 998-999.<br>依據著作權法第 46 、 52 、 65 條合理使用 |
| 25 | Theorem (R-S Theorem 4.7): The student-proposing DA is strategy- | | A.E. Roth and Marilda Sotomayer, Two-Sided Matching, Cambridge UP, 1992, pp.90.<br>依據著作權法第 46 、 52 、 65 條合理使用 |