National Taiwan University
Material Mechanics

Chapter Seven

Analysis of Stress and Strain

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TOPIC

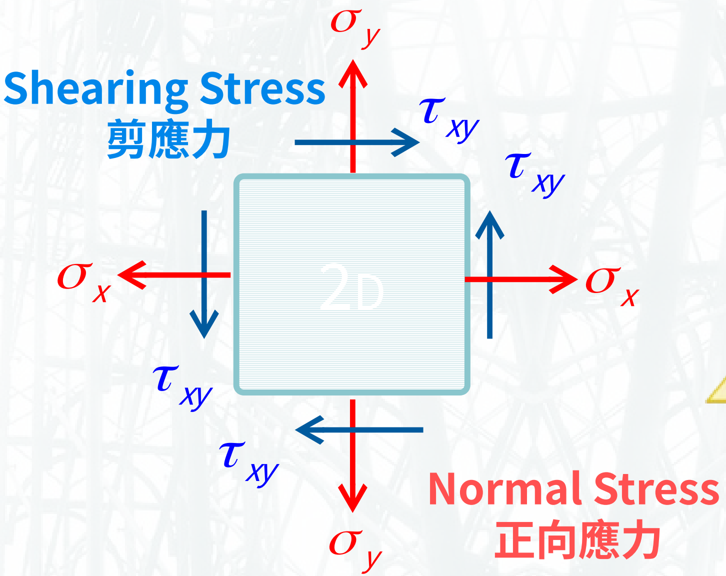
Catalog of Chap.7

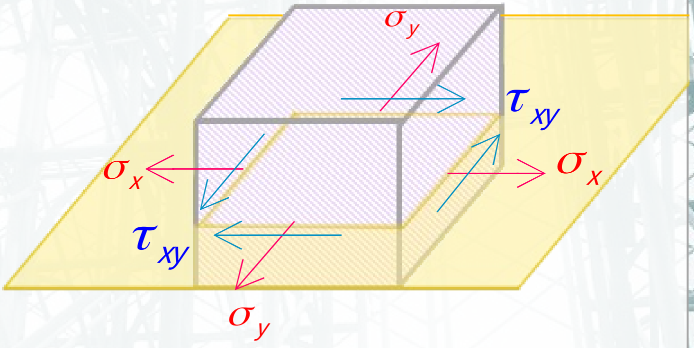
KEYPOINT

當一組梁的位知數超過平衡方程式所能解時，為靜不定，需要加入其他條件求解。

Plane stress

$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum M = 0$$





TOPIC

Plane stress transformation

KEYPOINT

2D 平面上，有兩個方向的正向力、一剪力。
3D 則有三個方向的正向力與三個方向的剪力。

Plane stress : Three stress components 三項 σ_x , σ_y , τ_{xy}

From plane stress to general 3D stress

TOPIC

Plane stress transformation

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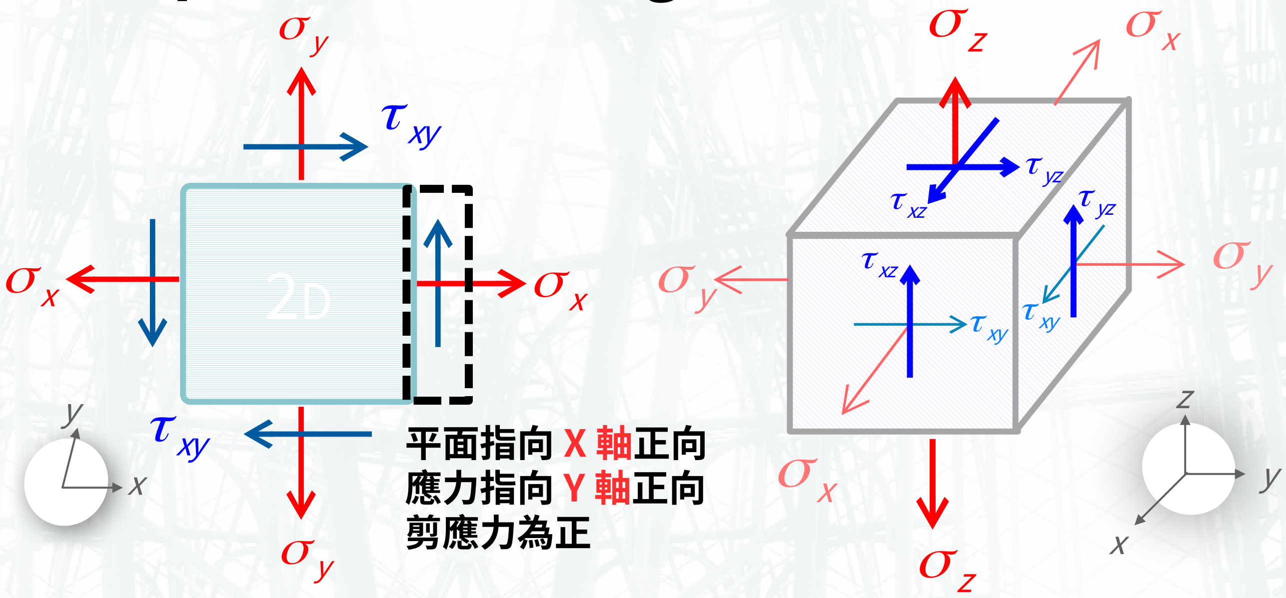
2D 平面上，有兩個方向的正向力、一剪力。
3D 則有三個方向的正向力與三個方向的剪力。



General stress : Six components 六項 σ_x , σ_y , σ_z, τ_{xy} , τ_{yz} , τ_{xz}

正向應力方向 ：向平面外作用為正

剪應力方向：平面方向與平面上剪應力皆指座標軸正 (負) 向者為正

Plane stress application

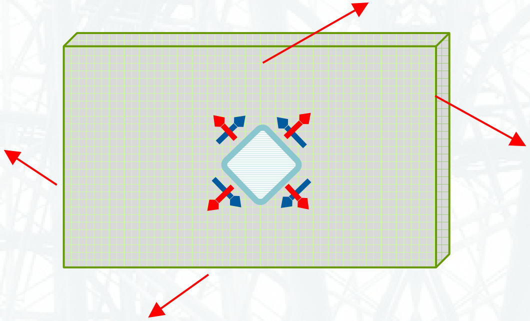
Plane stress : the stress components along a certain direction are all zero.

TOPIC

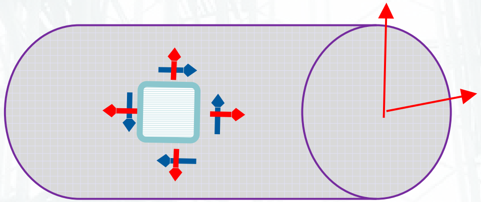
Plane stress transformation

KEYPOINT

當關於某一方向的所有應力為 0 ，我們就可以直接用平面應力來解釋物體的力學行為。



A thin plate subjected to forces acting in the mid-plane of the plate. **如板、牆**



at any point of the surface not subjected to an external force. **如結構桿件**

Plane stress transformation

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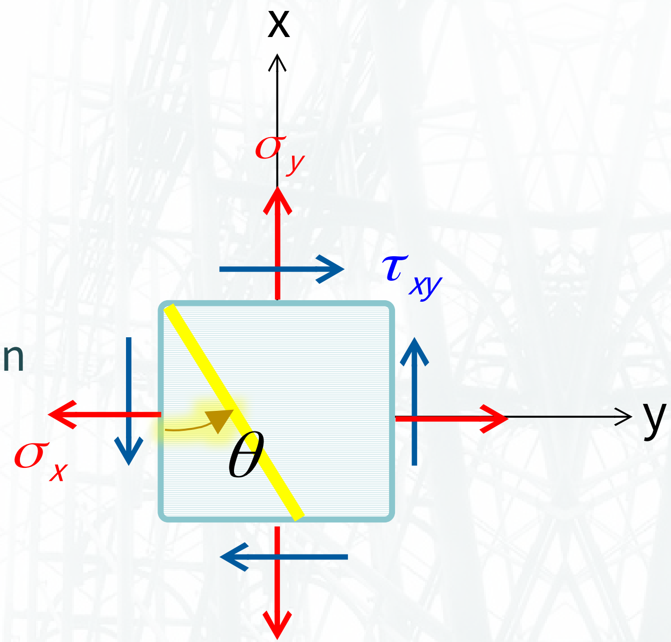
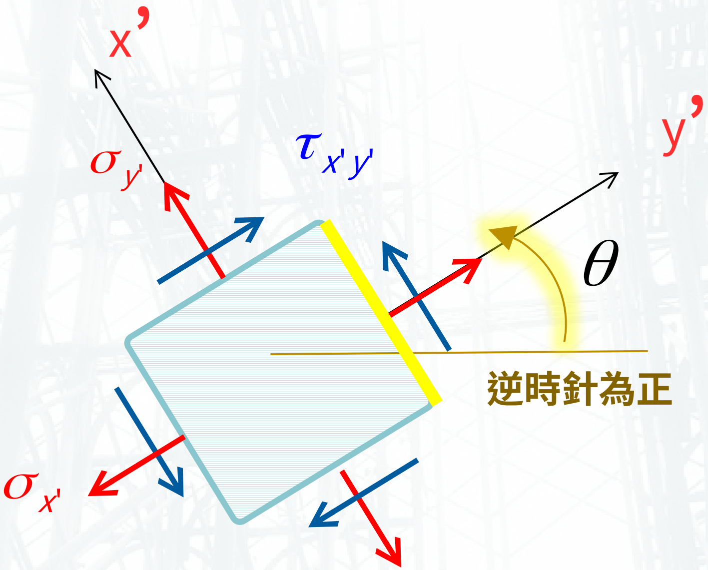
Plane stress transformation

KEYPOINT

以**逆時針為正**旋轉物體以及座標軸方向，找出對應該平面的平面應力大小。



斜面上的應力能利用水平位置切出的三角楔形計算



斜面上的正向應力與剪應力在旋轉後已經改變

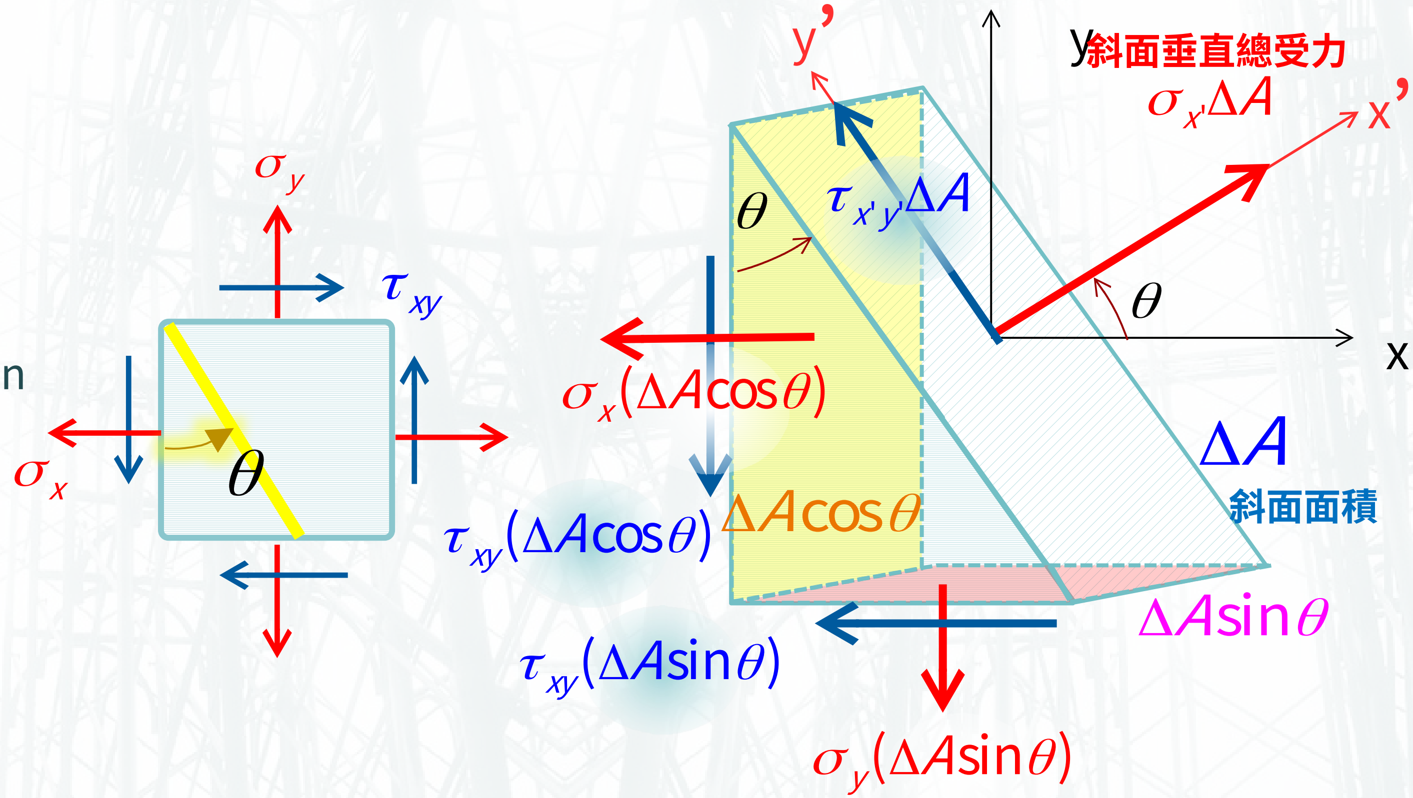
Plane stress transformation

TOPIC

Plane stress transformation

KEYPOINT

利用該斜面切割立面，形成四角錐。計算角錐力平衡，找到斜面受力大小。



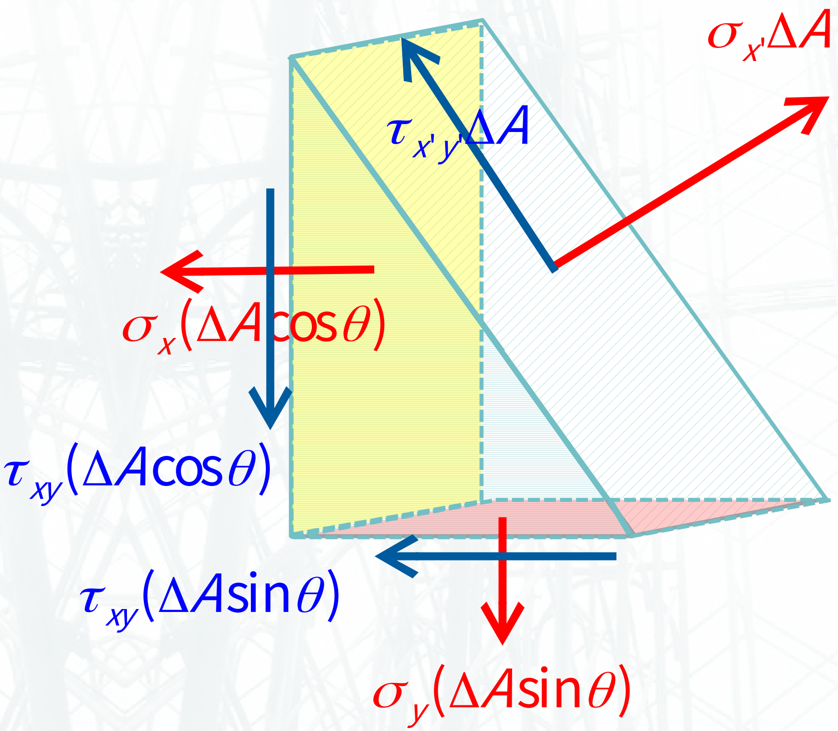
Plane stress transformation

TOPIC

Plane stress transformation

$$\sum F_{x'} = 0$$

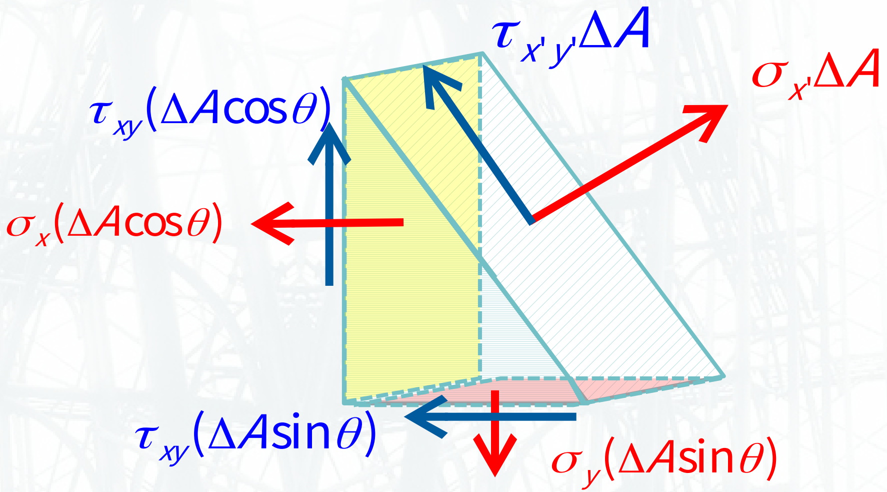
$$\sum F_{y'} = 0$$



KEYPOINT

計算角錐力平衡，由水平、垂直合力，找到斜面受力大小。

Plane stress transformation



$$\sum F_{x'}=0 \quad \sigma_{x'}\Delta A-\sigma_x(\Delta A\cos\theta)\cos\theta-\tau_{xy}(\Delta A\cos\theta)\sin\theta$$
$$-\sigma_y(\Delta A\sin\theta)\sin\theta-\tau_{xy}(\Delta A\sin\theta)\cos\theta=0$$

$$\sum F_{y'}=0 \quad \tau_{x'y'}\Delta A+\sigma_x(\Delta A\cos\theta)\sin\theta-\tau_{xy}(\Delta A\cos\theta)\cos\theta$$
$$-\sigma_y(\Delta A\sin\theta)\cos\theta+\tau_{xy}(\Delta A\sin\theta)\sin\theta=0$$

TOPIC

Plane stress transformation

KEYPOINT

計算角錐力平衡，由水平、垂直合力，找到斜面受力大小。

Plane stress transformation

$$\sum F_{x'} = 0 \quad \sigma_{x'}\cancel{\Delta A} - \sigma_x(\cancel{\Delta A}\cos\theta)\cos\theta - \tau_{xy}(\cancel{\Delta A}\cos\theta)\sin\theta$$
$$- \sigma_y(\cancel{\Delta A}\sin\theta)\sin\theta - \tau_{xy}(\cancel{\Delta A}\sin\theta)\cos\theta = 0$$

$$\sum F_{y'} = 0 \quad \tau_{x'y'}\cancel{\Delta A} + \sigma_x(\cancel{\Delta A}\cos\theta)\sin\theta - \tau_{xy}(\cancel{\Delta A}\cos\theta)\cos\theta$$
$$- \sigma_y(\cancel{\Delta A}\sin\theta)\cos\theta + \tau_{xy}(\cancel{\Delta A}\sin\theta)\sin\theta = 0$$

TOPIC

Plane stress transformation

$$\sin^2\theta + \cos^2\theta = 1$$
$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$\sigma_{x'} = \sigma_x\cos^2\theta + 2\tau_{xy}\sin\theta\cos\theta + \sigma_y\sin^2\theta$$
$$\tau_{x'y'} = (\sigma_y - \sigma_x)\sin\theta\cos\theta + \tau_{xy}\cos^2\theta - \tau_{xy}\sin^2\theta$$

KEYPOINT

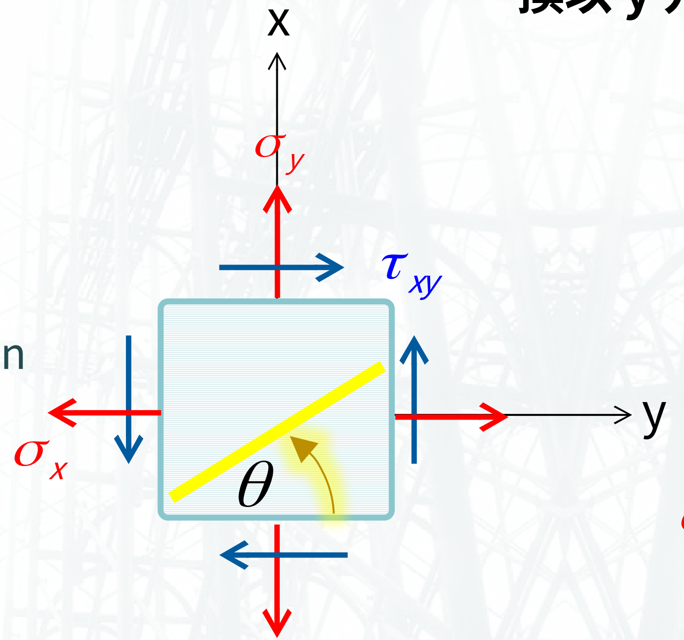
角錐力平衡方程式可以利用三角函數性質來化簡

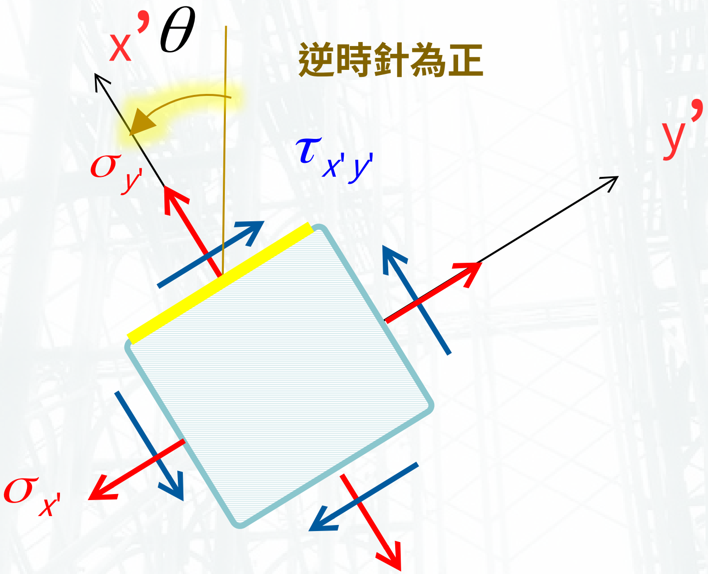
$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

應力旋轉 θ 角度後轉換公式

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

Plane stress transformation
換以 y 方向來分析





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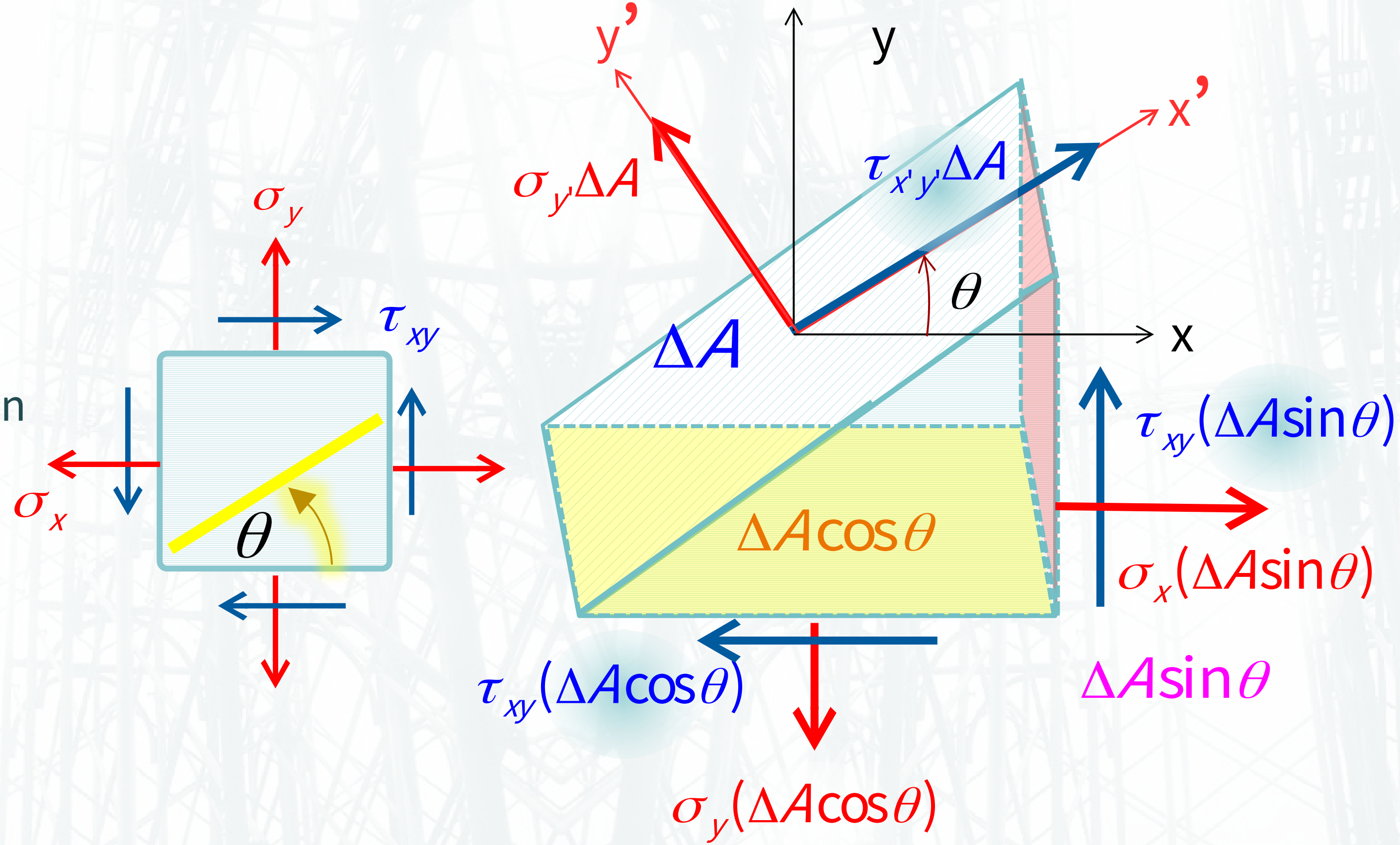
Plane stress transformation



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Plane stress transformation

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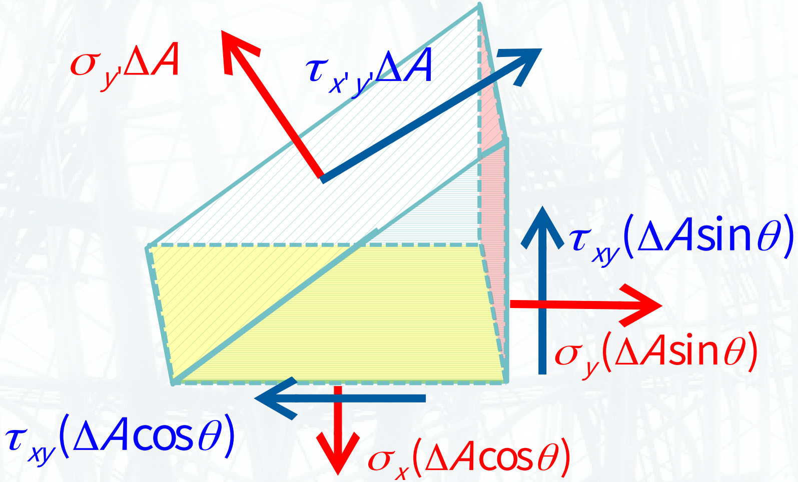
Plane stress transformation

TOPIC

Plane stress transformation



KEYPOINT

計算角錐力平衡，由水平、垂直合力，找到斜面受力大小。

$$\sum F_{y'} = 0 \quad \sigma_{y'}\Delta A - \sigma_y(\Delta A\cos\theta)\cos\theta + \tau_{xy}(\Delta A\cos\theta)\sin\theta - \sigma_x(\Delta A\sin\theta)\sin\theta + \tau_{xy}(\Delta A\sin\theta)\cos\theta = 0$$

$$\sum F_{x'} = 0 \quad \tau_{x'y'}\Delta A - \sigma_y(\Delta A\cos\theta)\sin\theta - \tau_{xy}(\Delta A\cos\theta)\cos\theta + \sigma_x(\Delta A\sin\theta)\cos\theta + \tau_{xy}(\Delta A\sin\theta)\sin\theta = 0$$

Plane stress transformation

$$\sum F_{y'} = 0 \quad \sigma_{y'}\Delta A - \sigma_y(\Delta A\cos\theta)\cos\theta + \tau_{xy}(\Delta A\cos\theta)\sin\theta - \sigma_x(\Delta A\sin\theta)\sin\theta + \tau_{xy}(\Delta A\sin\theta)\cos\theta = 0$$

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$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

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Plane stress transformation

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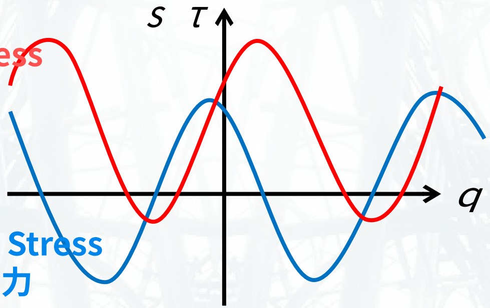
斜面應力是角度的函數



斜面應力是角度的函數

$$t = f(q)$$

$$s = f(q)$$

$$s_{x'}(2q + 180) = s_{y'}$$

帶入

$$s_{y¢} = \frac{s_x + s_y}{2} - \frac{s_x - s_y}{2}\cos 2q - t_{xy}\sin 2q$$

$$t_{x¢y¢} = -\frac{s_x - s_y}{2}\sin 2q + t_{xy}\cos 2q$$

$$s_{x¢} = \frac{s_x + s_y}{2} + \frac{s_x - s_y}{2}\cos 2q + t_{xy}\sin 2q$$

Matrix Form

旋轉矩陣來表示之前的應力角度換算公式

$$\underbrace{\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}}_{P^{-1}} \begin{bmatrix} \sigma_x & \tau_{xy} \\ \tau_{xy} & \sigma_y \end{bmatrix} \underbrace{\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}}_{P} = \begin{bmatrix} \sigma_{x'} & \tau_{x'y'} \\ \tau_{x'y'} & \sigma_{y'} \end{bmatrix}$$

$$\sigma_x + \sigma_y = \sigma_{x'} + \sigma_{y'}$$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

TOPIC

Plane stress transformation

KEYPOINT

斜面應力的函數可以用旋轉矩陣來直接表示。

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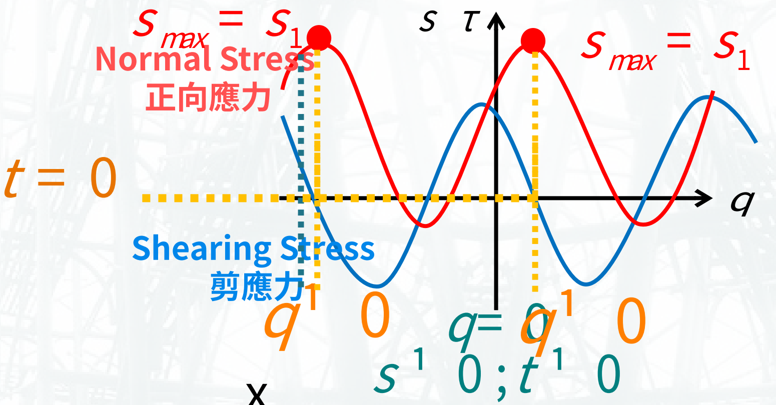
Chap.7

KEYPOINT

當一組梁的位知數超過平衡方程式所能解時，為靜不定，需要加入其他條件求解。

Principal stresses

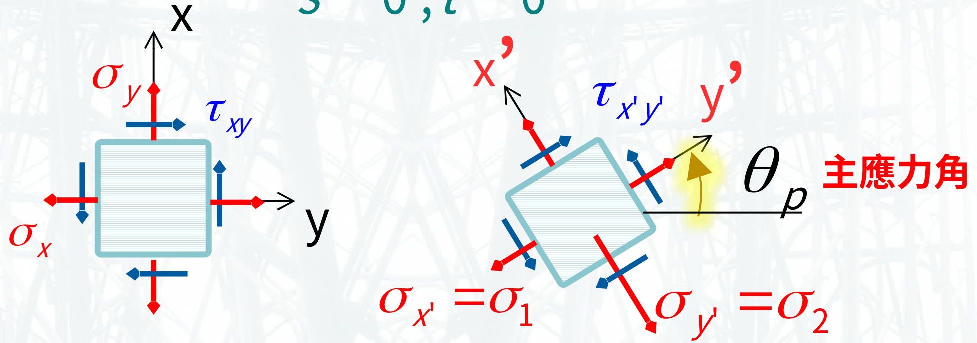


主應力面 s_1

1 具最大的正向應力 s_{max}

2 剪應力為零

3 主應力面旋轉後，新斜面對應新剪應力與正向應力值



$q = 0$ 題目情境物水平擺放之應力

傾斜一定角度
主應力面、剪力為 0

TOPIC

Principle stress and Max. shear stress in plane

KEYPOINT

當斜面上恰好沒有剪應力時，此時正向應力是最大的，稱為主應力。

Principal stresses

TOPIC

Principle stress and Max. shear stress in plane

KEYPOINT

主應力上，較大的正向應力為 σ_1，較小者為 σ_2

主應力與水平夾角 θp

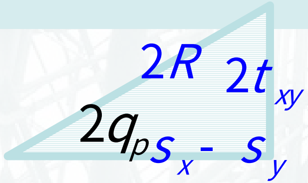
$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

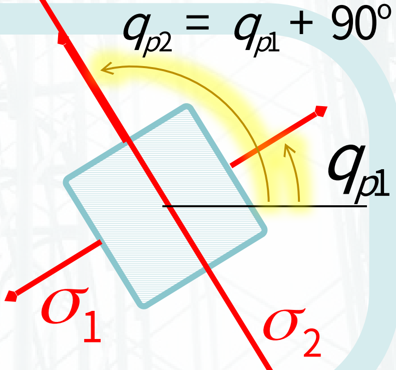
$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta = 0$$

主應力角

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$





$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \underbrace{\boxed{\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}}}_{R} \quad (\sigma_1 > \sigma_2)$$

不取絕對值

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Max. shear stresses in plane

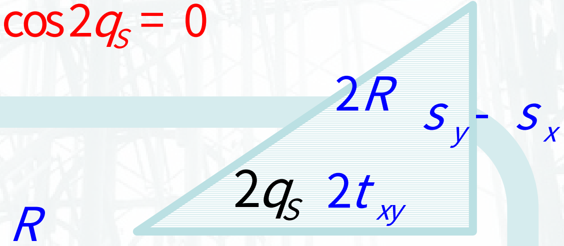
$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

最大剪應力角

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\frac{d\tau_{x'y'}}{d\theta} = -(\sigma_x - \sigma_y)\sin 2\theta_s + 2\tau_{xy}\cos 2\theta_s = 0$$



$$\tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$

$$\tau_{\max(in\text{-}plane)} = \boxed{\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}}$$

$$\sigma' = \sigma_{ave} = \frac{\sigma_x + \sigma_y}{2}$$

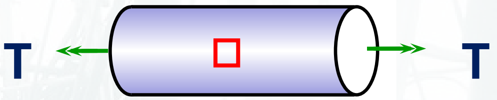
TOPIC

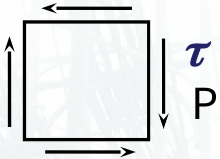
Principle stress and Max. shear stress in plane

KEYPOINT

剪應力最大值時，物體與水平面夾角為 θs 。且此時正向應力不一定為 0 。

Torque Example





$$\sigma_x = 0, \quad \sigma_y = 0, \quad \tau_{xy} = -\tau$$

Maximum in-plane shear stress

$$\tau_{\max(in-plane)} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \pm\tau$$

Average normal stress

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = 0$$

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Principle stress and Max. shear stress in plane

KEYPOINT

一物受扭矩，水平時該應力分布為正向應力=0 、只有剪應力。旋轉應力面後，可以找到新的正向應力和剪應力

Torque Example

Principle Stress $\sigma_x = 0, \quad \sigma_y = 0, \quad \tau_{xy} = -\tau$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 0 \pm \sqrt{(0)^2 + \tau^2} = \pm\tau$$

TOPIC

Principle stress and Max. shear stress in plane

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2(-\tau)}{0-0} = \infty \qquad \theta_p = 45^\circ, 135^\circ$$

Check 正負應力對應角度

KEYPOINT

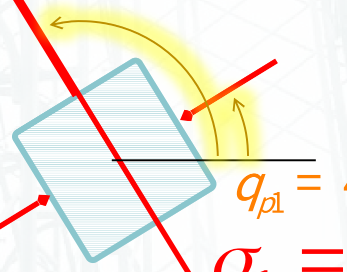
一物受扭矩，水平時該應力分布為正向應力=0 、只有剪應力。旋轉應力面後，可以找到新的正向應力和剪應力

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= 0 + 0\cos 90^\circ + (-\tau)\sin 90^\circ = -\tau$$

$$\theta_{p1} = 135^\circ \quad \theta_{p2} = 45^\circ$$

$q_{p2} = q_{p1} + 90^\circ = 135^\circ$



$q_{p1} = 45^\circ$

$\sigma_2 = -\tau$

$\sigma_1 = \tau$

Axial Force Example

$$\sigma_x = \sigma, \quad \sigma_y = 0, \quad \tau_{xy} = 0 \qquad \sigma_1 = \sigma, \quad \sigma_2 = 0$$

Maximum in-plane shear stress

$$\tau_{\max(in-plane)} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \pm\frac{\sigma}{2}$$

Average normal stress

$$\sigma' = \sigma_{ave} = \frac{\sigma}{2} = \sigma_1 = \sigma_2$$

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Principle stress and Max. shear stress in plane

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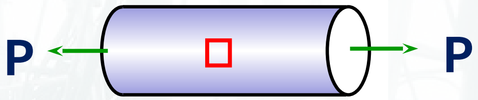
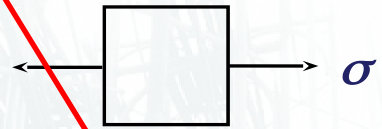
一物受軸向張力，只有軸向正向應力，剪應力和垂直向應力為 0 。旋轉應力面後，找到新的正向應力和剪應力

Axial Force Example





$$\tau_{\max(in\text{-}plane)} = \pm\frac{\sigma}{2} \qquad \sigma' = \sigma_{ave} = \frac{\sigma}{2} = \sigma_1 = \sigma_2$$

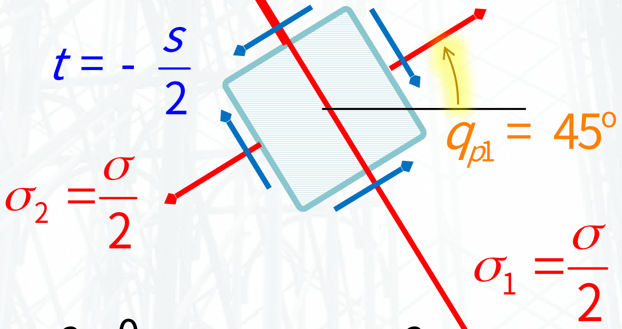
TOPIC

Principle stress and Max. shear stress in plane

Angle of Maximum in-plane shear stress

$$\tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}} = -\frac{\sigma - 0}{2\times 0}$$

$$\theta_{s1} = 45^\circ, \qquad \theta_{s2} = 135^\circ$$



KEYPOINT

一物受軸向張力，只有軸向正向應力，剪應力和垂直向應力為 0 。旋轉應力面後，找到新的正向應力和剪應力

Check 剪應力在此角度之正負值

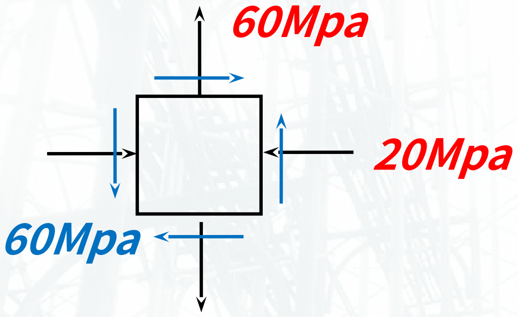
$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta = -\frac{\sigma - 0}{2}\sin 90^\circ + 0 = -\frac{\sigma}{2}$$

General Example

$\sigma_x = -20\ \mathrm{MPa}$

$\sigma_y = 90\ \mathrm{MPa}$

$\tau_{xy} = 60\ \mathrm{MPa}$



$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2(60)}{-20-90} = -1.09$$

$$2\theta_p = -47.49^\circ$$

$$\theta_p = -23.7^\circ,\ 66.3^\circ$$

Principle Stress

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{-20+90}{2} \pm \sqrt{\left(\frac{-20-90}{2}\right)^2 + (60)^2} = 35.0 \pm 81.4$$

TOPIC

Principle stress and Max. shear stress in plane

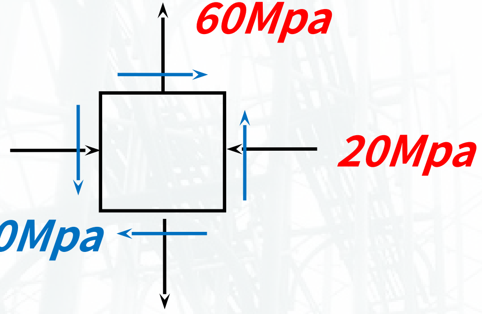
KEYPOINT

物體受應力的狀況隨旋轉 x 、 y 軸面，都能產生一組新的應力狀況。

General Example

$$\theta_p = -23.7^\circ,\quad 66.3^\circ$$

$$\sigma_{1,2} = 116.4, -46.4$$



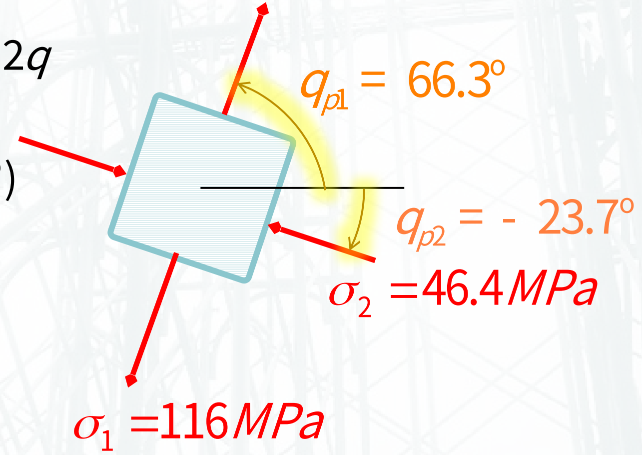
TOPIC

Principle stress and Max. shear stress in plane

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{-20+90}{2} + \frac{-20-90}{2}\cos 2(-23.7^\circ)$$

$$+ 60\sin 2(-23.7^\circ) = -46.4\ \mathrm{MPa}$$



KEYPOINT

物體受應力的狀況隨旋轉 x 、 y 軸面，都能產生一組新的應力狀況。

$$\theta_{p1} = 66.3^\circ,\quad \theta_{p2} = -23.7^\circ$$

General Example

Maximum in-plane shear stress

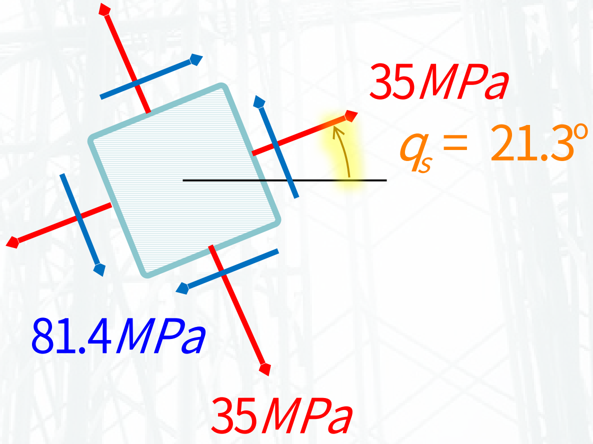
$$\theta_s = \theta_p + 45^\circ \qquad \theta_{p1} = 66.3^\circ, \quad \theta_{p2} = -23.7^\circ \qquad \theta_s = 21.3^\circ, \ 111.3^\circ$$

$$\tau_{\max(in-plane)} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{20 - 90}{2}\right)^2 + (60)^2} = \pm 81.4 \text{ MPa}$$

$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\left(\frac{20 - 90}{2}\right)\sin 2(21.3^\circ)$$

$$+ 60\cos 2(21.3^\circ) = 81.4 \text{ MPa}$$



Average normal stress

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = \frac{-20 + 90}{2} = 35 \text{MPa}$$

TOPIC

Principle stress and Max. shear stress in plane

KEYPOINT

物體受應力的狀況隨旋轉 x 、 y 軸面，都能產生一組新的應力狀況。

TOPIC

Catalog of Chap.7

KEYPOINT

當一組梁的位知數超過平衡方程式所能解時，為靜不定，需要加入其他條件求解。

Principal stresses

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta = 0$$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

TOPIC

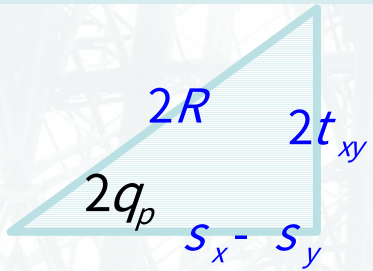
Mohr’s Circle-Plane Stress

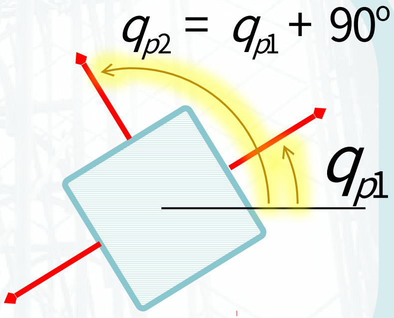
KEYPOINT

主應力上，較大的正向應力為 σ_1，較小者為 σ_2

主應力與水平夾角 θp

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$





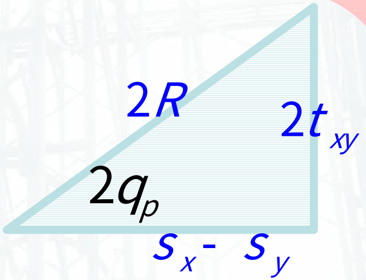
$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \underbrace{\boxed{\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}}}_{R} \quad (\sigma_1 > \sigma_2)$$

Principal stresses

$$\tau_{x'y'}^2 = \left(-\frac{\sigma_x-\sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta\right)^2$$

$$\left(\sigma_{x'} - \frac{\sigma_x+\sigma_y}{2}\right)^2 = \left(\frac{\sigma_x-\sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta\right)^2$$

$$R = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2} \qquad \sigma_{ave} = \frac{\sigma_x+\sigma_y}{2}$$



$$(\sigma_{x'} - \sigma_{ave})^2 + \tau_{x'y'}^2 = R^2$$

TOPIC

Mohr's Circle-Plane Stress

KEYPOINT

從主應力公式推導莫耳圓的半徑定義。

Mohr's Circle

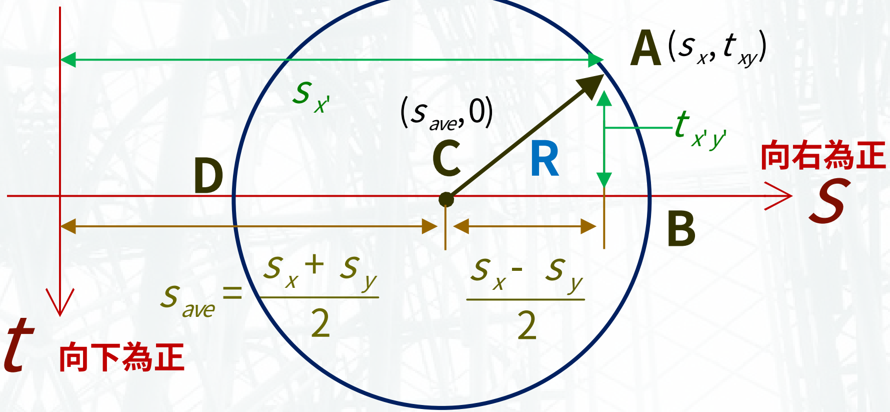
$$(\sigma_{x'} - \sigma_{ave})^2 + \tau_{x'y'}^2 = R^2$$

莫耳圓半徑

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

莫耳圓圓心

$C\ (\sigma_{ave}, 0)$



主應力代表位置

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \quad (\sigma_1 > \sigma_2)$$

$\sigma_1 = \sigma_{ave} + R,\ \sigma_2 = \sigma_{ave} - R,\quad$ B $(\sigma_1, 0)$, D $(\sigma_2, 0)$

TOPIC

Mohr's Circle-Plane Stress

KEYPOINT

莫耳圓的定義。橫軸為正向應力，縱軸為剪應力，圓上每一點都代表物體某角度斜面的力分布。

Mohr's Circle and Principle Stress

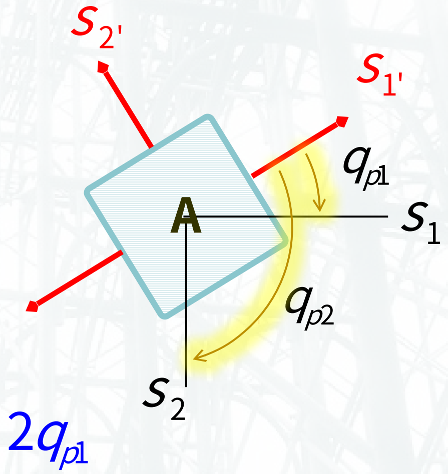
$$(s_{x'} - s_{ave})^2 + t_{x'y'}^2 = R^2$$

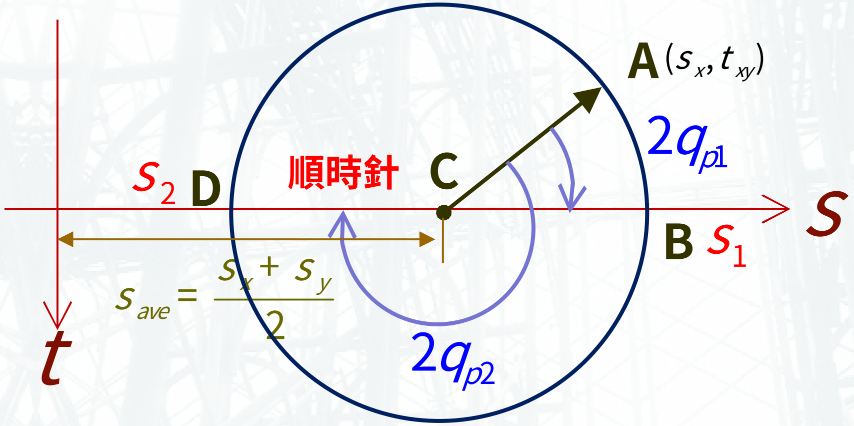
TOPIC

Mohr's Circle-Plane Stress

KEYPOINT

莫耳圓上相異點的旋轉角度是實際物體旋轉的兩倍。可以應用到主應力位置和最大剪應力位置上。





A 點應力位置旋轉到主應力面上角度

σ_2 : A 位置順時針旋轉 θ_{P2}

σ_1 : A 位置順時針旋轉 θ_{P1}

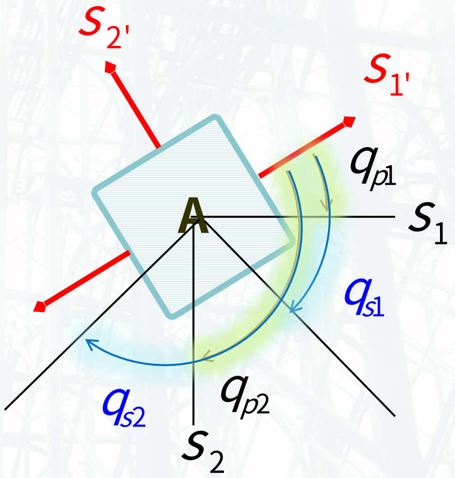
$$q_{p2} = q_{p1} + 90^\circ$$

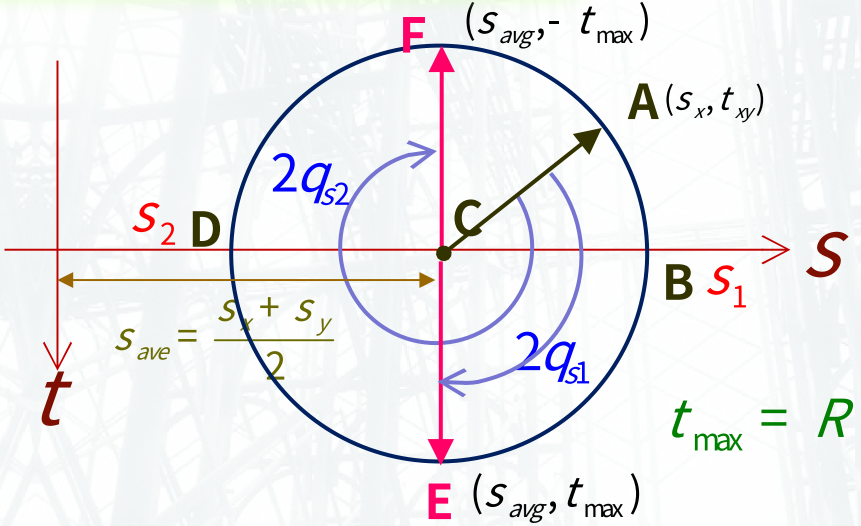
Mohr's Circle and Max in-plane shear stress

$$\theta_{s1} = \theta_{p1} + 45^{\circ} \qquad \theta_{s2} = \theta_{s1} + 90^{\circ}$$

TOPIC

Mohr's Circle-Plane Stress





KEYPOINT

莫耳圓上相異點的旋轉角度是實際物體旋轉的兩倍。可以應用到主應力位置和最大剪應力位置上。

A 點應力位置旋轉到最大剪應力面角度

$-\tau_{max}$: A 位置順時針旋轉 θ_{s2}

τ_{max} : A 位置順時針旋轉 θ_{s1}

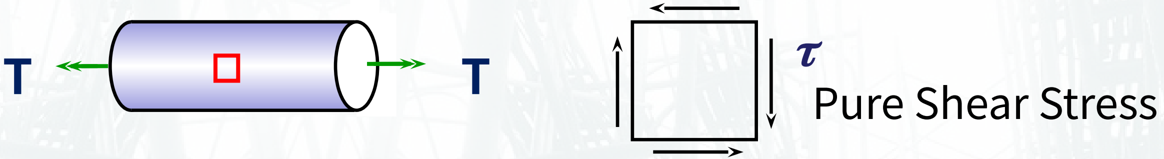
Torque Example

TOPIC

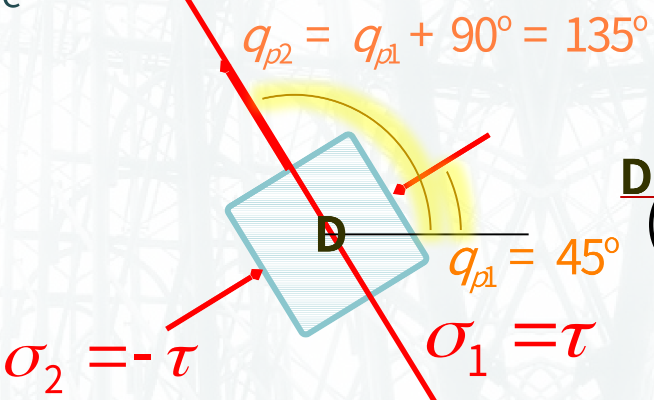
Mohr's Circle-Plane Stress

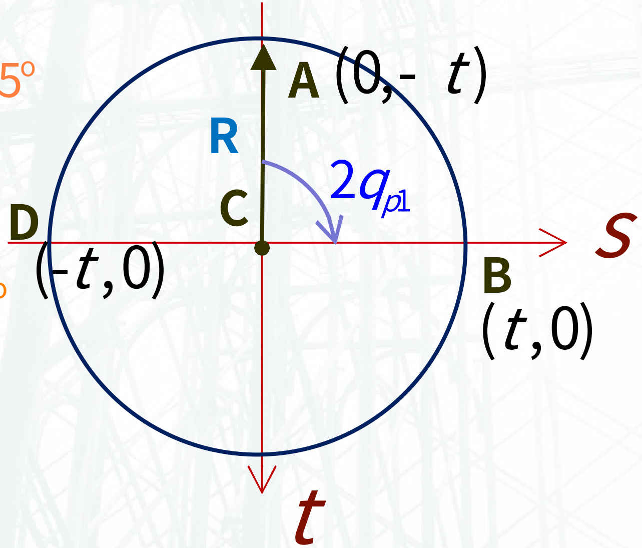
KEYPOINT

受扭矩的莫耳圓。



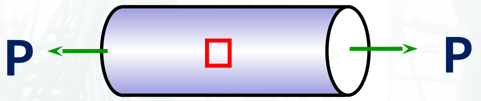
$$\sigma_x = 0, \quad \sigma_y = 0, \quad \tau_{xy} = -\tau$$

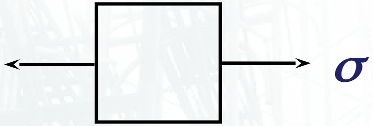




Axial Force Example

$s_x = s, \quad s_y = 0, \quad t_{xy} = 0 \qquad s_1 = s, \quad s_2 = 0$



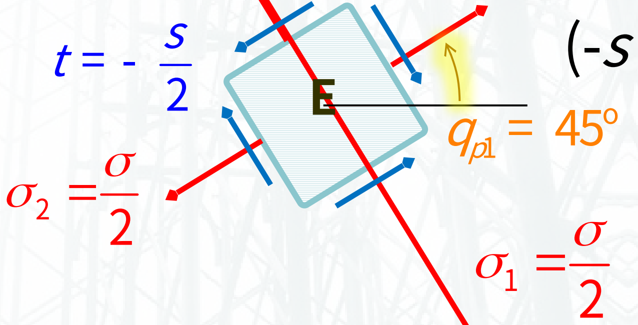


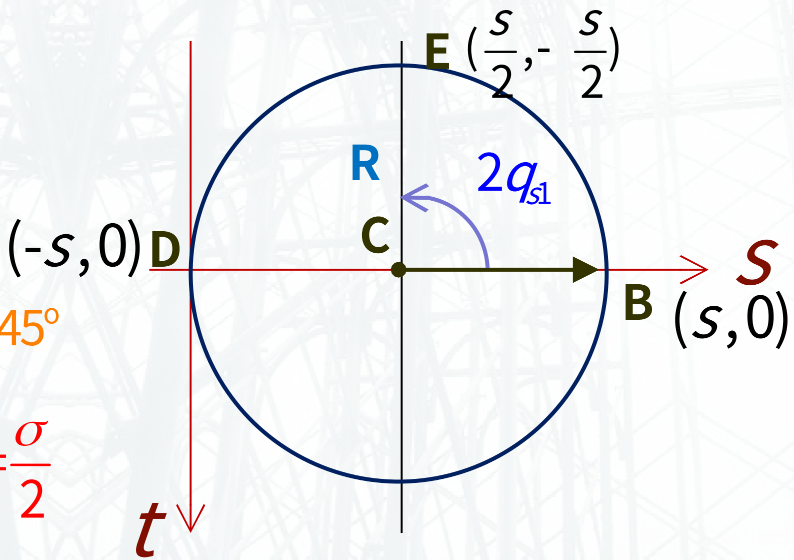
TOPIC

Mohr's Circle-Plane Stress

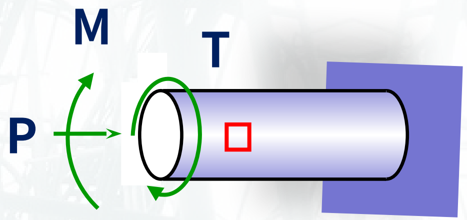
KEYPOINT

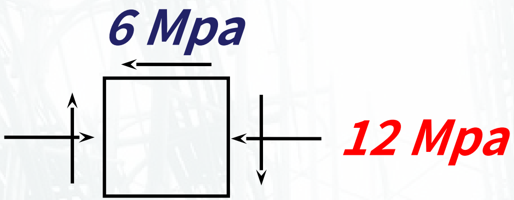
受軸力的莫耳圓。





General Example 1





TOPIC

Mohr's Circle-Plane Stress

KEYPOINT

利用莫耳圓快速找到主應力面和最大剪應力面的資訊。

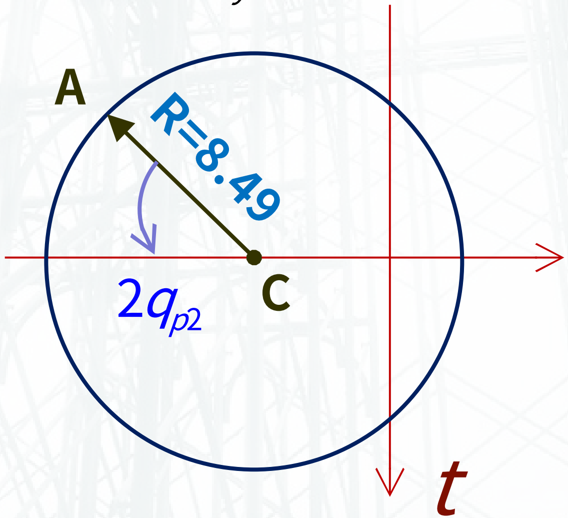
$$\sigma_x = -12\,\mathrm{MPa}, \quad \sigma_y = 0, \quad \tau_{xy} = -6\,\mathrm{MPa}$$

$$\sigma_{ave} = \frac{-12+0}{2} = -6\,\mathrm{MPa}$$

A $(-12, -6)$

C $(-6, 0)$

$$R = \sqrt{(12-6)^2 + (6)^2} = 8.49\,\mathrm{MPa}$$



General Example 1

$$\sigma_1 = \sigma_{ave} + R = -6 + 8.49 = 2.49\,\text{MPa}$$

$$\sigma_2 = \sigma_{ave} - R = -6 - 8.49 = -14.5\,\text{MPa}$$

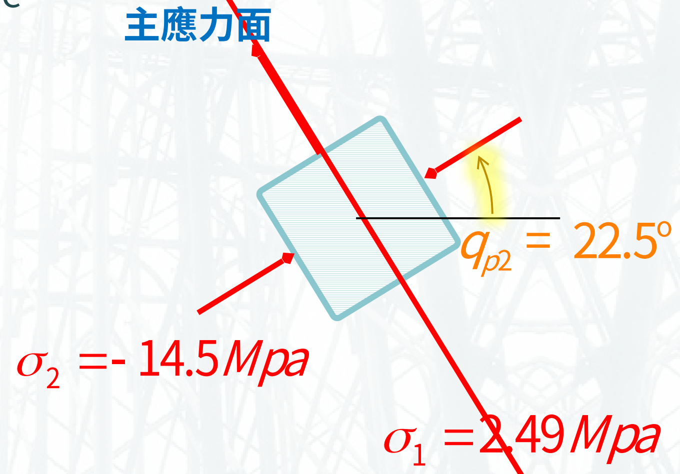
$$\tan 2\theta_{p2} = \frac{6}{12-6} = 1 \quad 2\theta_{p2} = 45^\circ \quad \theta_{p1} = 112.5^\circ \quad \theta_{p2} = 22.5^\circ$$

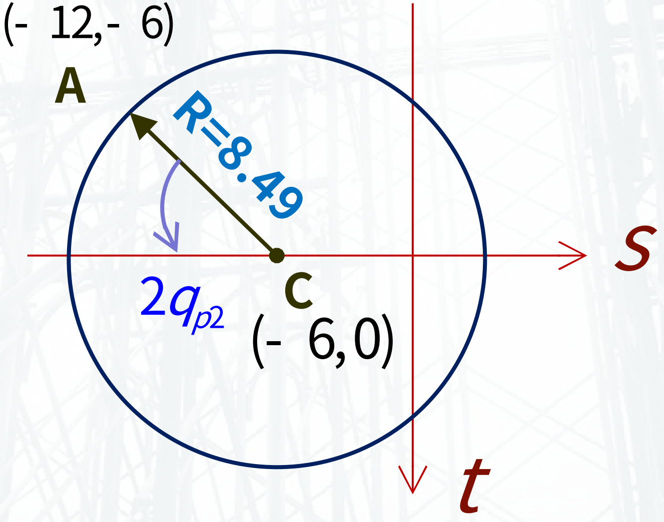
TOPIC

Mohr's Circle-Plane Stress

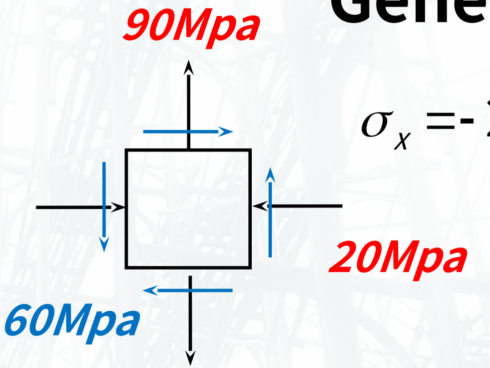
KEYPOINT

利用莫耳圓快速找到主應力面和最大剪應力面的資訊。





General Example 2



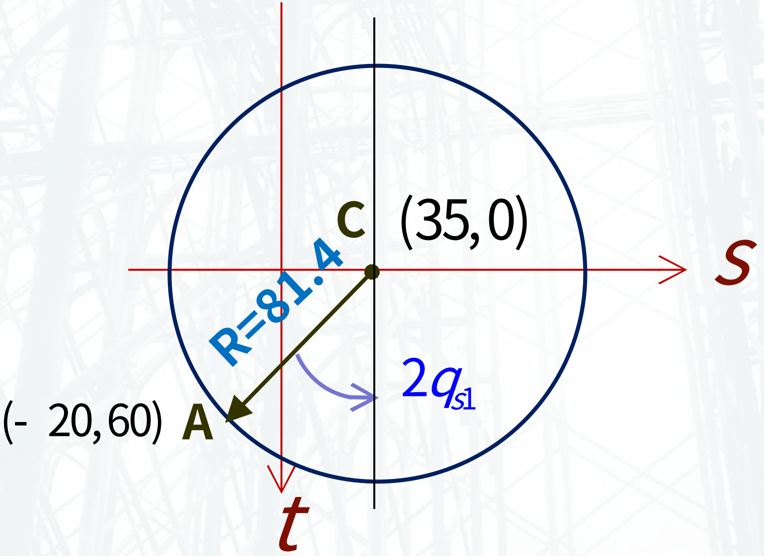
$$\sigma_x = -20\,\mathrm{MPa}, \quad \sigma_y = 90\,\mathrm{MPa}, \quad \tau_{xy} = 60\,\mathrm{MPa}$$

$$\sigma_{ave} = \frac{-20+90}{2} = 35\,\mathrm{MPa}$$

A $(-20,60)$ C $(35,0)$

$$R = \sqrt{(60)^2 + (55)^2}$$

$$= 81.4\ \mathrm{MPa}$$



TOPIC

Mohr’s Circle-Plane Stress

KEYPOINT

利用莫耳圓快速找到主應力面和最大剪應力面的資訊。

General Example 2

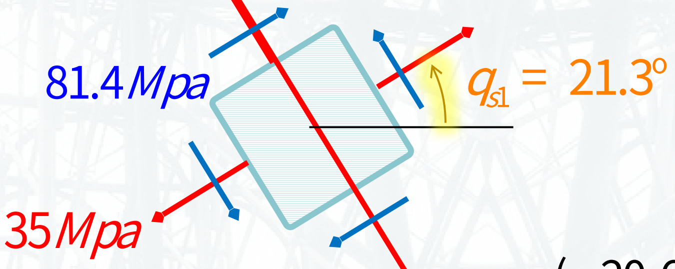
$$\tau_{max(in-plane)} = 81.4\ MPa \qquad 2\theta_{s1} = 42.5^{\circ}$$

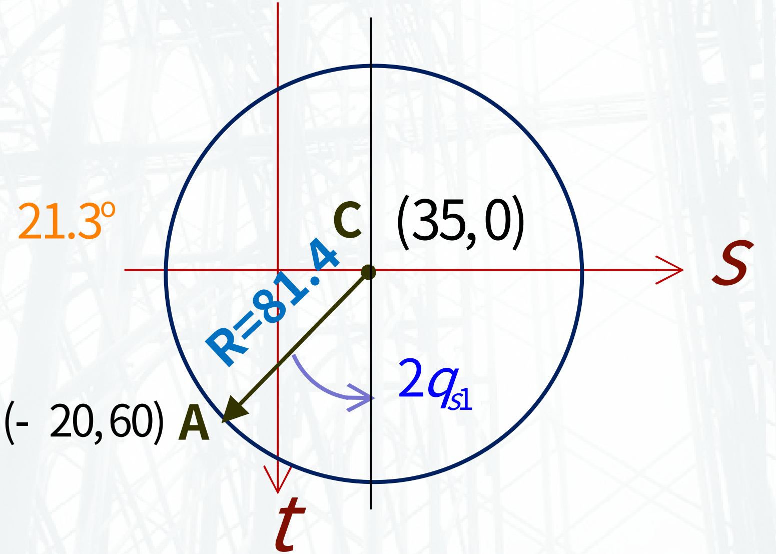
$$\tan 2\theta_{s1} = \frac{20+35}{60} = 0.917 \qquad \theta_{s1} = 21.3^{\circ} \quad \theta_{s2} = 111.3^{\circ}$$

TOPIC

Mohr's Circle-Plane Stress

最大剪應力面





KEYPOINT

利用莫耳圓快速找到主應力面和最大剪應力面的資訊。

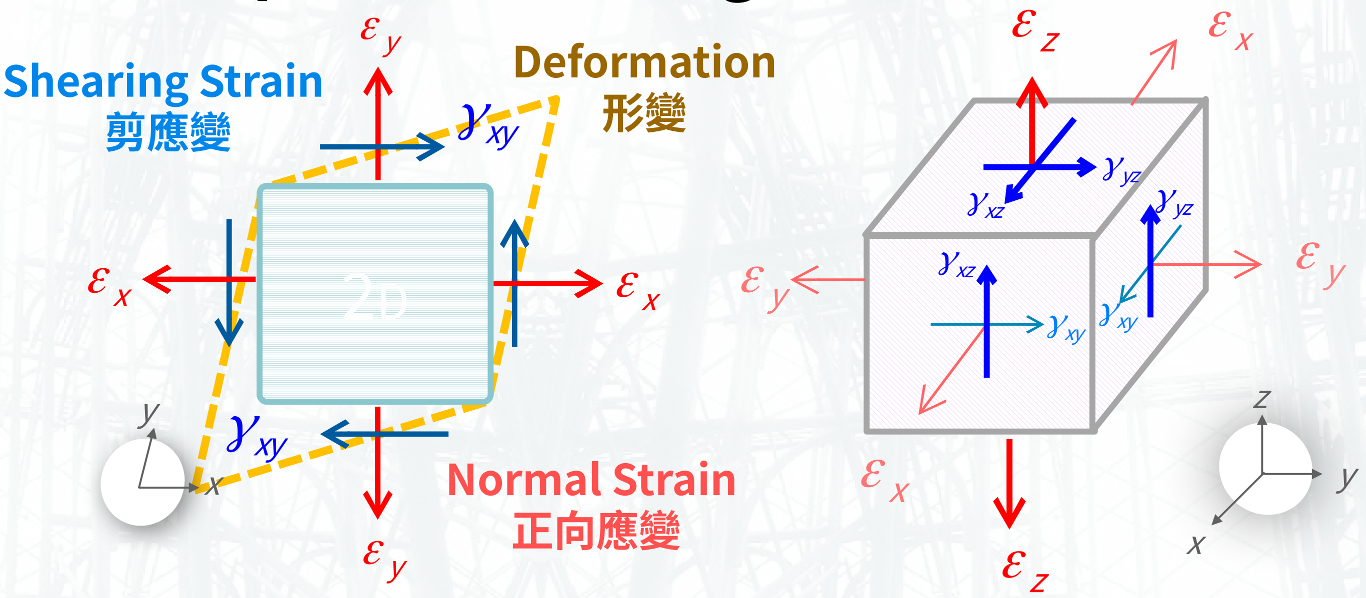
TOPIC

Catalog
of
Chap.7

KEYPOINT

當一組梁的位知數超過平衡方程式所能解時，為靜不定，需要加入其他條件求解。

From plane strain to general 3D strain



TOPIC

Plane strain transformation

KEYPOINT

2D 平面上，有兩個方向的正向應變、一剪應變。
3D 則有三個方向的正向應變與三個方向的剪應變。

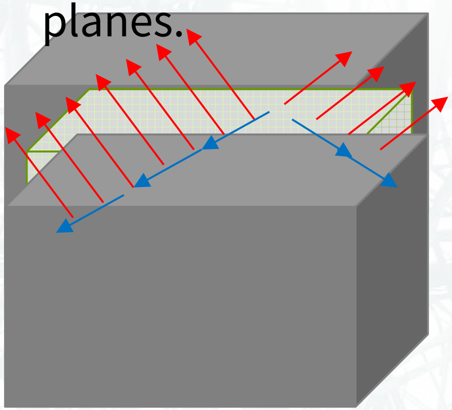
General strain : Six components 六項$_x$, ε_y, ε_z, γ_{xy}, γ_{yz}, γ_{xz}

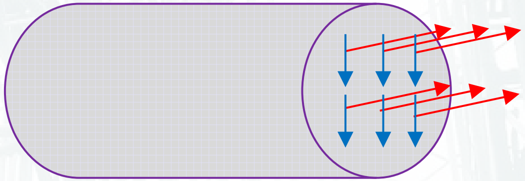
$\gamma_{xy} = \gamma_{yx}, \gamma_{yz} = \gamma_{zy}, \gamma_{zx} = \gamma_{xz}$

$t = Gg \qquad s = Ee$

Plane strain application

Plane stress : deformations of the material in parallel planes are the same in each of those planes.





Restrained laterally by smooth, rigid and fixed supports. **另兩方向被固定，無法隨意變形。**

TOPIC

Plane stress transformation

KEYPOINT

當關於某一方向的所有應變為 0 ，我們就可以直接用平面應變來解釋物體的變形行為。

Plane strain transformation

TOPIC

Plane strain transformation

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

KEYPOINT

應力旋轉的轉換方程式，
可以用相同的推導方法。

★ 正向應力 σ 改成
正向應變 ε

★ 剪應力 τ 改成
1/2 剪應變 γ/2

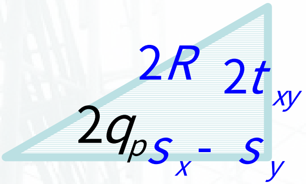
$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$\frac{\gamma_{x'y'}}{2} = -\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$\varepsilon_{y'} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

Principle strain

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$



主應力面
主應力角

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \quad (\sigma_1 > \sigma_2)$$

TOPIC

Plane strain transformation

KEYPOINT

應力旋轉的轉換方程式，
可以用相同的推導方法。

★ 正向應力☒改成
正向應變 ε

★ 剪應力 τ 改成
1/2 剪應變 γ/2

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y}$$

$$\varepsilon_{1,2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

Max shear strain in plane

TOPIC

Plane strain transformation

KEYPOINT

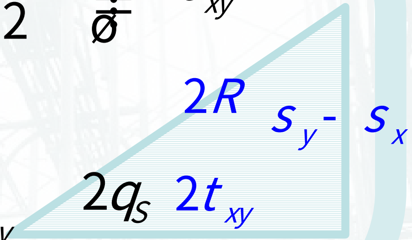
應力旋轉的轉換方程式，
可以用相同的推導方法。

最大剪應力面
最大剪應力角

$$\tau_{\max(in-plane)} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$

$$\sigma' = \sigma_{ave} = \frac{\sigma_x + \sigma_y}{2}$$



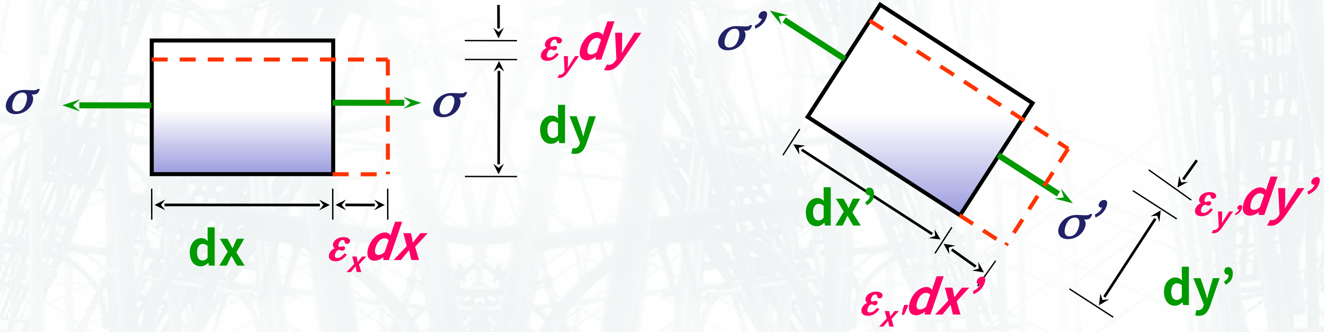
★ 正向應力□改成
正向應變 ε

★ 剪應力 τ 改成
1/2 剪應變 γ/2

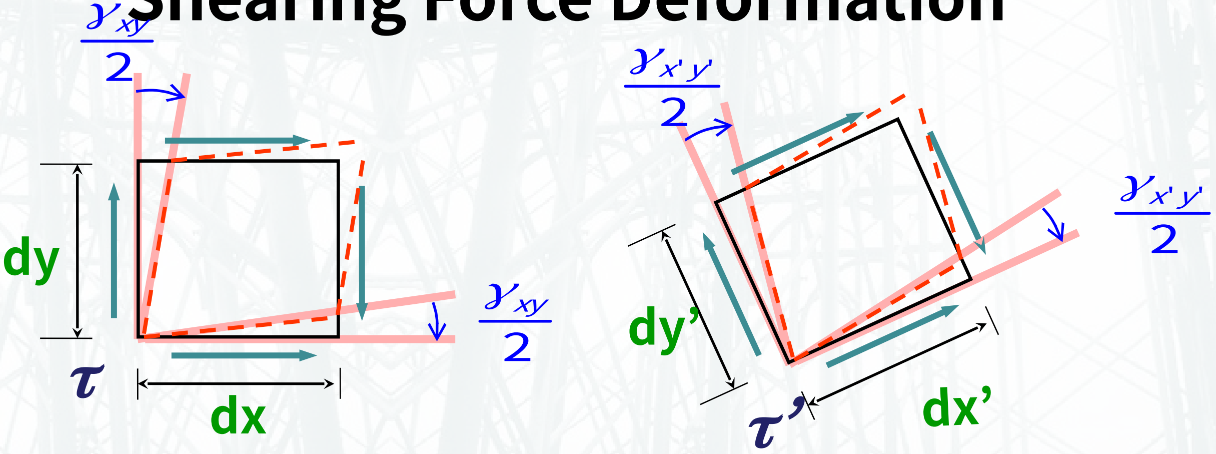
$$\frac{\gamma_{\max(in-plane)}}{2} = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\tan 2\theta_s = -\frac{\varepsilon_x - \varepsilon_y}{\gamma_{xy}} \qquad \varepsilon_{ave} = \frac{\varepsilon_x + \varepsilon_y}{2}$$

Axial Force Deformation



Shearing Force Deformation



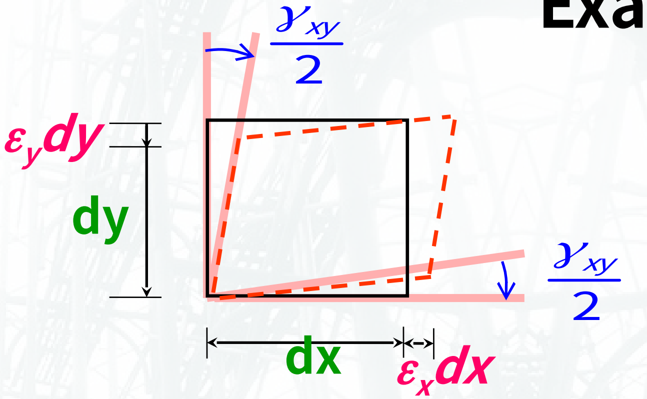
TOPIC

Plane strain transformation

KEYPOINT

物體變形時應變的物理意義。

Example 1



$\varepsilon_x = 500\ (10^{-6})$

$\varepsilon_y = -300\ (10^{-6})$

$\gamma_{xy} = 200(10^{-6})$

Determine strains **clockwise 30°** from the original position.

TOPIC

Plane strain transformation

KEYPOINT

物體應變可用旋轉公式轉換不同角度的應變值。

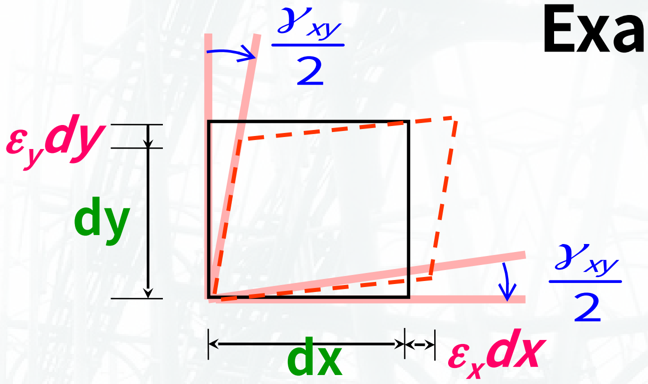
$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta = \frac{500 + (-300)}{2} \times 10^{-6}$$

$$+ \frac{500 - (-300)}{2} \times 10^{-6} \times \cos(2(-30^\circ)) + \frac{200}{2} \times 10^{-6} \times \sin(2(-30^\circ)) = 213(10^{-6})$$

$$\frac{\gamma_{x'y'}}{2} = -\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta = -\frac{500 - (-300)}{2} \times 10^{-6} \times \sin(2(-30^\circ))$$

$$+ \frac{200}{2} \times 10^{-6} \times \cos(2(-30^\circ)) \qquad \gamma_{x'y'} = 793(10^{-6})$$

Example 1



$\varepsilon_x = 500\ (10^{-6})$

$\varepsilon_y = -300\ (10^{-6})$

$\gamma_{xy} = 200 (10^{-6})$

TOPIC

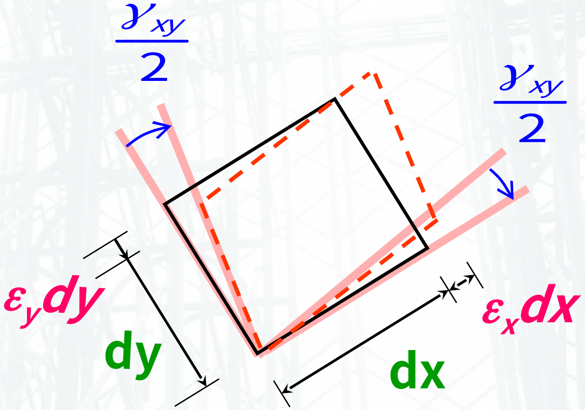
Plane strain transformation

KEYPOINT

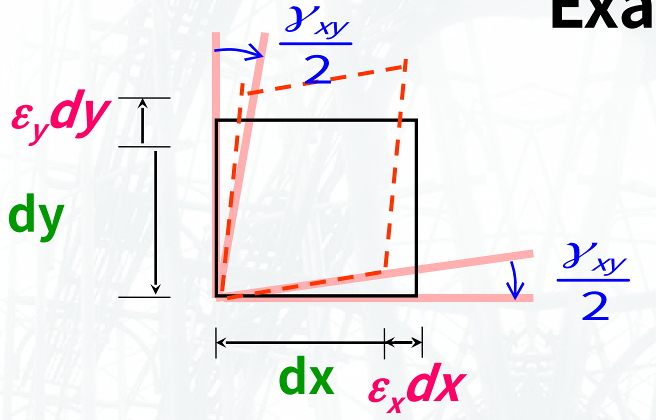
物體應變可用旋轉公式轉換不同角度的應變值。

$$\varepsilon_{y'} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \frac{500 + (-300)}{2} \times 10^{-6} - \frac{500 - (-300)}{2} \times 10^{-6} \times \cos(2(-30^\circ)) - \frac{200}{2} \times 10^{-6} \times \sin(2(-30^\circ)) = -13.4(10^{-6})$$



Example 2



$\varepsilon_x = -350(10^{-6})$

$\varepsilon_y = 200(10^{-6})$

$\gamma_{xy} = 80(10^{-6})$

Determine the **principal strain** and associated orientation.

TOPIC

Plane strain transformation

KEYPOINT

物體應變可用旋轉公式轉換不同角度的應變值。

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{80(10^{-6})}{(-350-200)(10^{-6})}$$

$$2\theta_p = -8.28^\circ \quad \text{and } 171.72^\circ$$

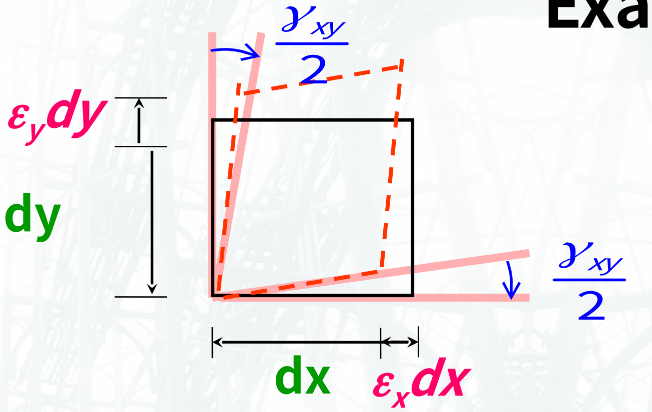
$$\theta_p = -4.14^\circ \quad \text{and} \quad 85.9^\circ$$

$$\varepsilon_{1,2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = \frac{(-350+200)(10^{-6})}{2} \pm \sqrt{\left(\frac{-350-200}{2}\right)^2 + \left(\frac{80}{2}\right)^2} \times 10^{-6}$$

$$= -75.0(10^{-6}) \pm 277.9(10^{-6})$$

$$\varepsilon_1 = 203(10^{-6}) \qquad \varepsilon_2 = -353(10^{-6})$$

Example 2



$\varepsilon_x = -350(10^{-6})$

$\varepsilon_y = 200(10^{-6})$

$\gamma_{xy} = 80(10^{-6})$

Determine the **principal strain** and associated orientation.

TOPIC

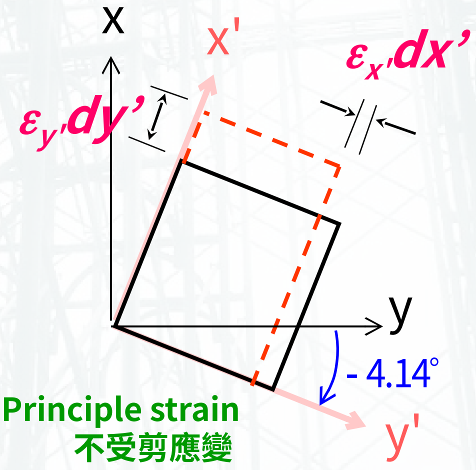
Plane strain transformation

KEYPOINT

物體應變可用旋轉公式轉換不同角度的應變值。

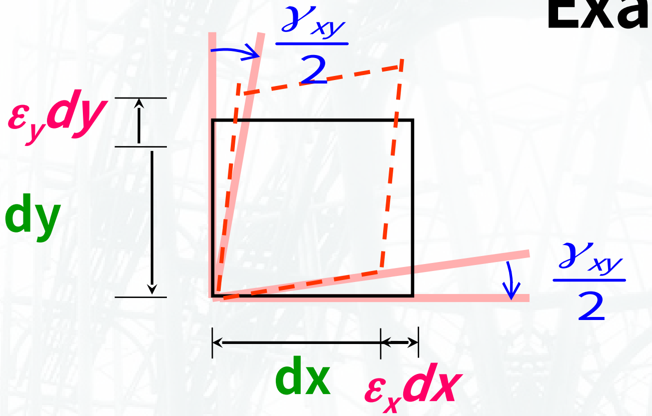
$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \frac{-350+200}{2} \times 10^{-6} + \frac{-350-200}{2} \times 10^{-6} \times \cos(2(-4.14^\circ)) + \frac{80}{2} \times 10^{-6} \times \sin(2(-4.14^\circ)) = -353(10^{-6})$$



Principle strain
不受剪應變

Example 2



$\varepsilon_x = -350(10^{-6})$

$\varepsilon_y = 200(10^{-6})$

$\gamma_{xy} = 80(10^{-6})$

Determine the **maximum in-plane shear strain** and associated orientation.

TOPIC

Plane strain transformation

KEYPOINT

物體應變可用旋轉公式轉換不同角度的應變值。

$$\tan 2\theta_s = -\frac{\varepsilon_x - \varepsilon_y}{\gamma_{xy}} = -\frac{(-350-200)(10^{-6})}{(80)(10^{-6})}$$

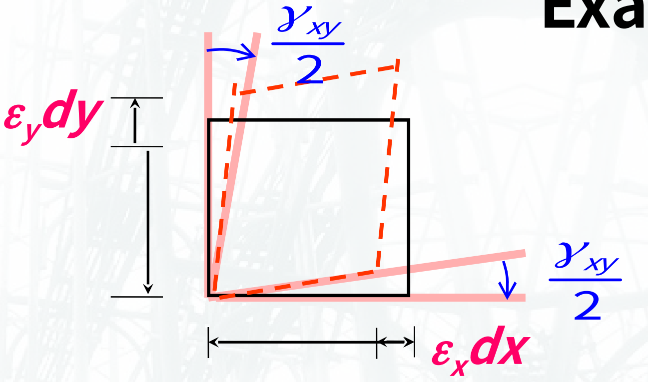
$2\theta_s = 81.72^\circ$ and 261.72

$\theta_s = 40.9^\circ$ and 130.9°

$$\frac{\gamma_{\max(in-plane)}}{2} = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = \sqrt{\left(\frac{-350-200}{2}\right)^2 + \left(\frac{80}{2}\right)^2} \times 10^{-6} = 277.9(10^{-6})$$

$$\gamma_{\max(in-plane)} = 556(10^{-6})$$

Example 2



$\varepsilon_x = -350(10^{-6})$

$\varepsilon_y = 200(10^{-6})$

$\gamma_{xy} = 80(10^{-6})$

Determine **maximum in-plane shear strain** and associated orientation.

TOPIC

Plane strain transformation

KEYPOINT

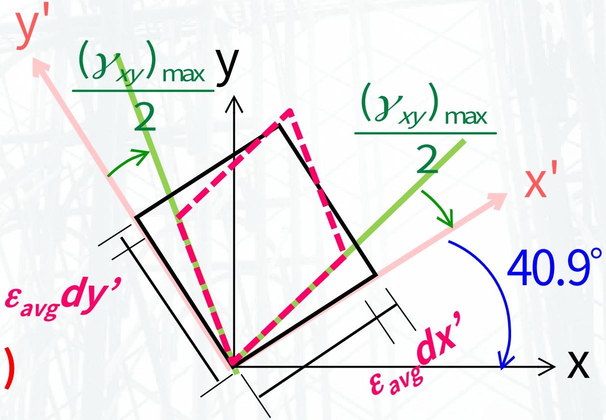
物體應變可用旋轉公式轉換不同角度的應變值。

$$\frac{\gamma_{x'y'}}{2} = -\frac{\varepsilon_x - \varepsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta =$$

$$-\frac{-350-200}{2} \times 10^{-6} \times \sin(2(40.9^\circ))$$

$$+\frac{80}{2} \times 10^{-6} \times \cos(2(40.9^\circ))$$

$$\gamma_{x'y'} = 556(10^{-6}) \qquad \varepsilon_{ave} = \frac{\varepsilon_x + \varepsilon_y}{2} = -75(10^{-6})$$



TOPIC

Catalog of Chap.7

KEYPOINT

當一組梁的位知數超過平衡方程式所能解時，為靜不定，需要加入其他條件求解。

Mohr's Circle

$$(\varepsilon_x - \varepsilon_{ave})^2 + \left(\frac{\gamma_{xy}}{2}\right)^2 = R^2$$

TOPIC

Mohr's Circle-Plane Strain

KEYPOINT

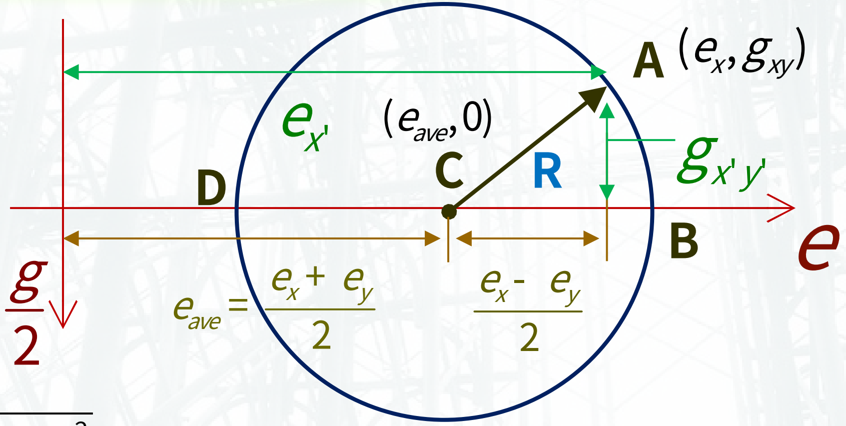
莫耳圓的定義。橫軸為正向應變，縱軸為剪應變，圓上每一點都代表物體某角度斜面的力分布。

莫耳圓半徑

$$R = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

莫耳圓圓心

$$C\ (\varepsilon_{avg}, 0)$$



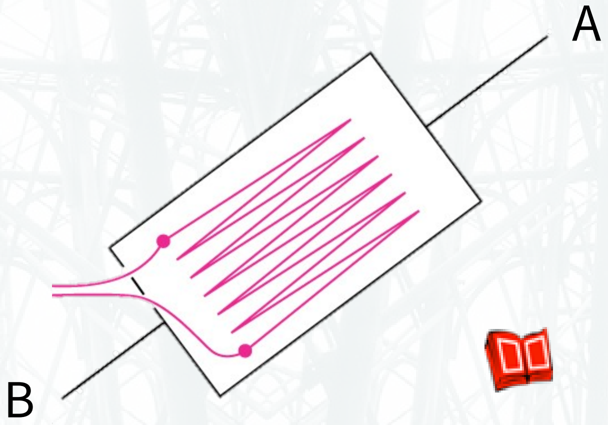
主應力代表位置

$$\varepsilon_{1,2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} \quad (\varepsilon_1 > \varepsilon_2)$$

$$\varepsilon_1 = \varepsilon_{ave} + R,\quad \varepsilon_2 = \varepsilon_{ave} - R,\quad B\ (\varepsilon_1, 0),\ D\ (\varepsilon_2, 0)$$

Strain Rosettes

- Strain gauge 應變計 indicate normal strain on the plane 平面
  上正向應變 through electrical-resistance 電阻 .
- Strain rosette → three electrical-resistance gauges.



TOPIC

Mohr's Circle-Plane Strain

KEYPOINT

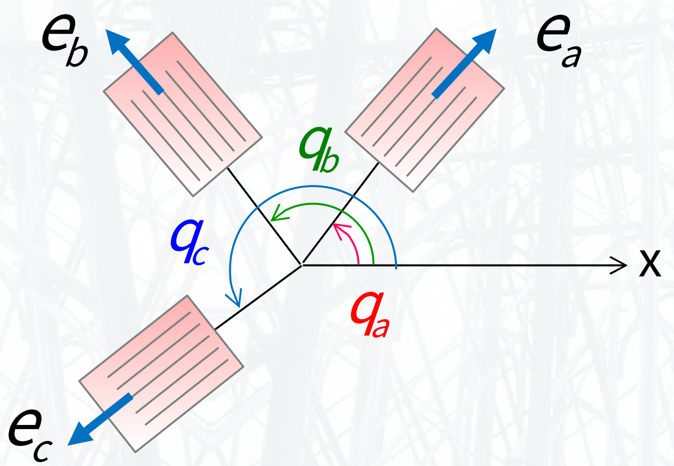
應變計的運用

General Strain Rosettes

$$e_a = e_x \cos^2 q_a + e_y \sin^2 q_a + g_{xy} \sin q_a \cos q_a$$

$$e_b = e_x \cos^2 q_b + e_y \sin^2 q_b + g_{xy} \sin q_b \cos q_b$$

$$e_c = e_x \cos^2 q_c + e_y \sin^2 q_c + g_{xy} \sin q_c \cos q_c$$



TOPIC

Mohr's Circle-Plane Strain

KEYPOINT

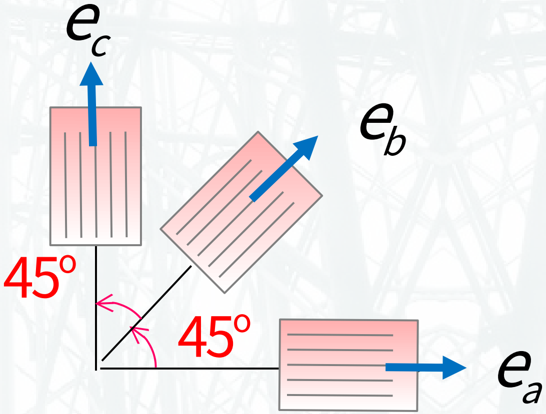
應變計的運用

Special Angle Strain Rosettes

$$e_a = e_x \underbrace{\cos^2 0}_{1} \cancel{+ e_y \sin^2 0 + g_{xy} \sin 0 \cos 0}$$

$$e_b = e_x \underbrace{\cos^2 45}_{0.5} + e_y \underbrace{\sin^2 45}_{0.5} + g_{xy} \underbrace{\sin 45 \cos 45}_{0.5}$$

$$e_c = \cancel{e_x \cos^2 90} + e_y \underbrace{\sin^2 90}_{1} \cancel{+ g_{xy} \sin 90 \cos 90}$$



$$\varepsilon_x = \varepsilon_a$$

$$\varepsilon_y = \varepsilon_c$$

$$\gamma_{xy} = 2\varepsilon_b - (\varepsilon_a + \varepsilon_c)$$

TOPIC

Mohr's Circle-Plane Strain

KEYPOINT

應變計的運用，夾 45 度角。

Special Angle Strain Rosettes

$$e_a = e_x \underbrace{\cos^2 0}_{1} + \cancel{e_y \sin^2 0 + g_{xy} \sin 0 \cos 0}$$

$$e_b = e_x \underbrace{\cos^2 60}_{0.25} + e_y \underbrace{\sin^2 60}_{0.75} + g_{xy} \underbrace{\sin 60 \cos 60}_{\frac{\sqrt{3}}{4}}$$

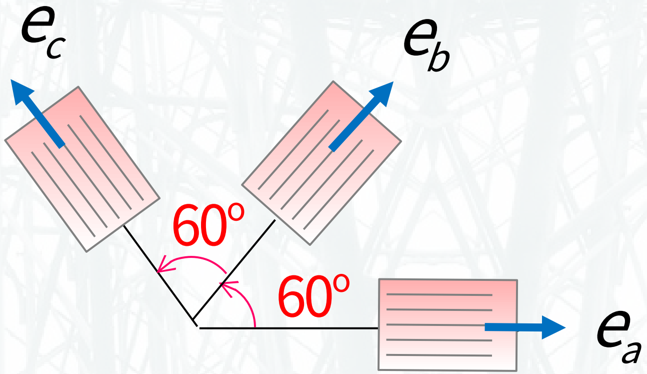
$$e_c = e_x \underbrace{\cos^2 120}_{0.25} + e_y \underbrace{\sin^2 120}_{0.75} + g_{xy} \underbrace{\sin 120 \cos 120}_{\frac{\sqrt{3}}{4}}$$

TOPIC

Mohr’s Circle-Plane Strain

KEYPOINT

應變計的運用，夾 60 度角。

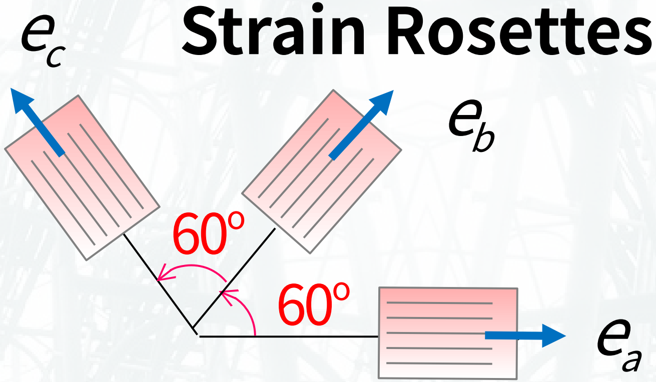


$$\varepsilon_x = \varepsilon_a$$

$$\varepsilon_y = \frac{1}{3}(2\varepsilon_b + 2\varepsilon_c - \varepsilon_a)$$

$$\gamma_{xy} = \frac{2}{\sqrt{3}}(\varepsilon_b - \varepsilon_c)$$

Strain Rosettes Example



$\varepsilon_a = 60(10^{-6})$

$\varepsilon_b = 135(10^{-6})$

$\varepsilon_c = 246(10^{-6})$

Determine **in-plane principal strains** and the directions

$\theta_a = 0^\circ,\ \theta_b = 60^\circ,\ \theta_c = 120^\circ$

TOPIC

Mohr's Circle-Plane Strain

KEYPOINT

應變計的運用，夾 60 度角。

解方程式

$$60(10^{-6}) = e_x\cos^2 0^\circ + e_y\sin^2 0^\circ + g_{xy}\sin 0^\circ\cos 0^\circ = e_x$$

$$135(10^{-6}) = e_x\cos^2 60^\circ + e_y\sin^2 60^\circ + g_{xy}\sin 60^\circ\cos 60^\circ$$

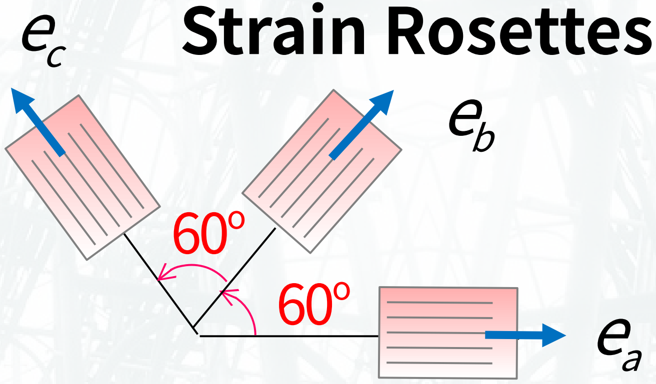
$$= 0.25e_x + 0.75e_y + 0.433g_{xy}$$

$$246(10^{-6}) = e_x\cos^2 120^\circ + e_y\sin^2 120^\circ + g_{xy}\sin 120^\circ\cos 120^\circ$$

$$= 0.25e_x + 0.75e_y - 0.433g_{xy}$$

$$\varepsilon_x = 60(10^{-6}),\quad \varepsilon_y = 246(10^{-6}),\quad \gamma_{xy} = -149(10^{-6})$$

Strain Rosettes Example



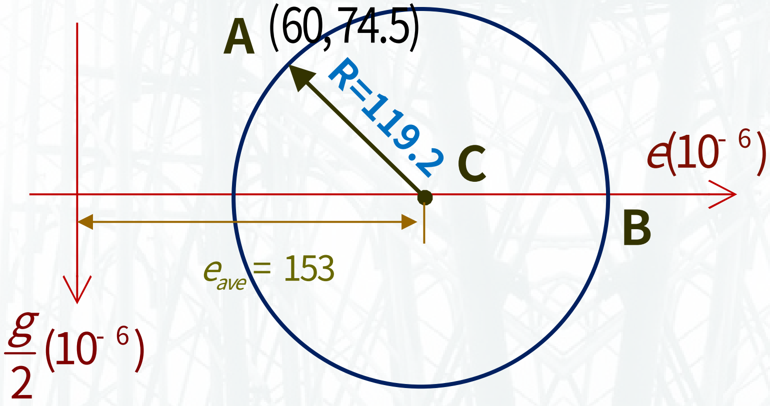
$\varepsilon_a = 60(10^{-6})$

$\varepsilon_b = 135(10^{-6})$

$\varepsilon_c = 246(10^{-6})$

Determine **in-plane principal strains** and the directions

$\varepsilon_x = 60(10^{-6}),\quad \varepsilon_y = 246(10^{-6}),\quad \gamma_{xy} = -149(10^{-6})$



$A(60(10^{-6}),\ -74.5(10^{-6}))$

$e_{ave} = 153(10^{-6})$

$R = \sqrt{(153-60)^2 + (74.5)^2} \times 10^{-6}$

$= 119.2(10^{-6})$

TOPIC

Mohr's Circle-Plane Strain

KEYPOINT

應變計的運用，夾 60 度角。

Strain Rosettes Example

$$\varepsilon_1 = 153(10^{-6}) + 119.2(10^{-6}) = 272(10^{-6})$$

$$\varepsilon_2 = 153(10^{-6}) - 119.2(10^{-6}) = 33.8(10^{-6})$$

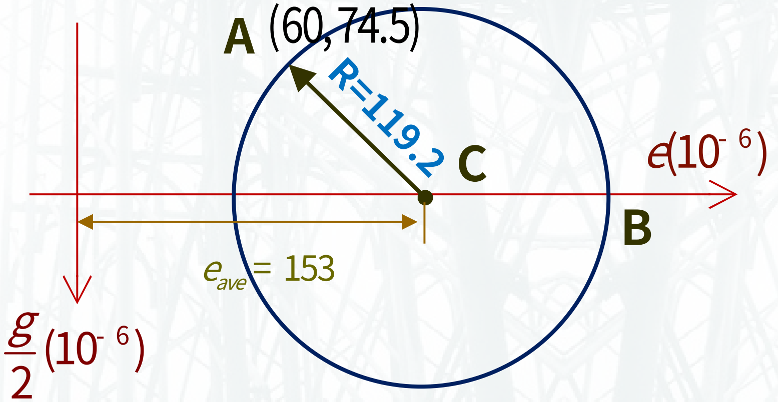
$$2\theta_p = \tan^{-1}\frac{74.5}{(153-60)} = 38.7^\circ \quad \theta_p = 19.3^\circ,\ 109.3^\circ$$

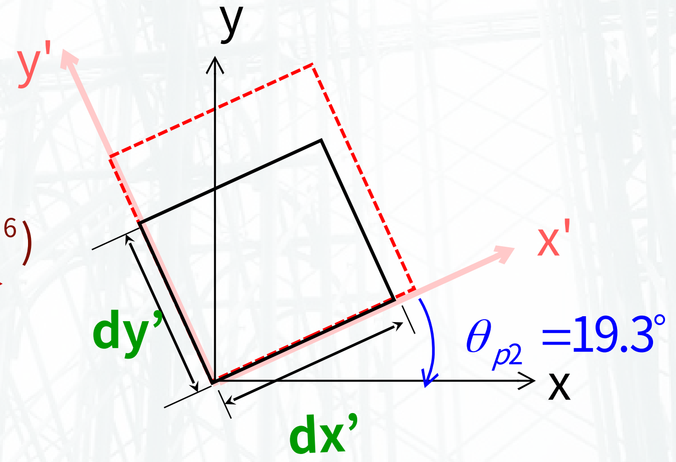
TOPIC

Mohr's Circle-Plane Strain

KEYPOINT

應變計的運用，夾 60 度角。





Material-Property Relationships

Hooke's Law

$$\sigma = E\,\varepsilon$$

$$\underset{(1)}{\varepsilon} = \frac{\delta\ (\mathrm{m})}{L\ (\mathrm{m})}$$

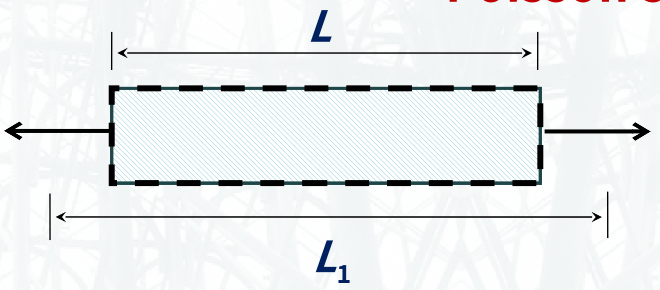
$$\underset{(\mathrm{N/m^2})}{\sigma} = \frac{P\ (\mathrm{N})}{A\ (\mathrm{m^2})}$$

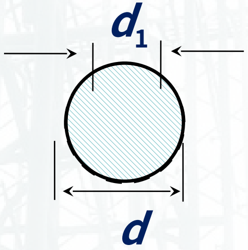
$$\Rightarrow \frac{P}{A}_{(\mathrm{N/m^2})} = E\frac{\delta}{L}$$

TOPIC

Mohr's Circle-Plane Strain

Poisson's Ratio





$$\nu = -\frac{\varepsilon'}{\varepsilon}$$

KEYPOINT

正向應變和應力間，可借由楊氏模數和柏松比轉換。剪應力和剪應變則由剪力模數轉換。

(橫向)受力 **Longitudinal strain**

$$\varepsilon = \frac{\delta}{L} = \frac{L_1 - L}{L}$$

(縱向)不受力 **Lateral strain**

$$\varepsilon' = \frac{\Delta d}{d} = \frac{d_1 - d}{d}$$

Material-Property Relationships

Using principle of superposition:

$$\varepsilon_x = \varepsilon_x^{'} + \varepsilon_x^{''} + \varepsilon_x^{'''} = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E}$$

三方向的施力對 x 方向應變的影響

Generalized Hooke's law

$$\varepsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)]$$

$$\varepsilon_y = \frac{1}{E}[\sigma_y - \nu(\sigma_x + \sigma_z)]$$

$$\varepsilon_z = \frac{1}{E}[\sigma_z - \nu(\sigma_x + \sigma_y)]$$

$$\gamma_{xy} = \frac{1}{G}\tau_{xy}, \quad \gamma_{yz} = \frac{1}{G}\tau_{yz}, \quad \gamma_{xz} = \frac{1}{G}\tau_{xz}, \quad G = \frac{E}{2(1+\nu)}$$

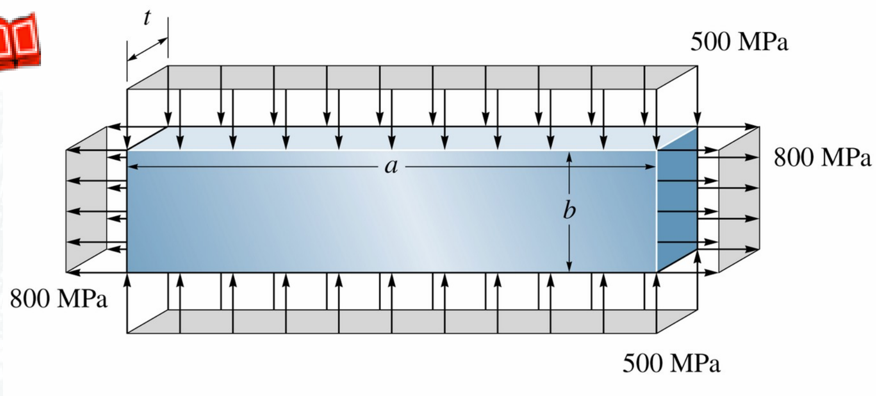
TOPIC

Mohr's Circle-Plane Strain

KEYPOINT

正向應變和應力間，可借由楊氏模數和柏松比轉換。剪應力和剪應變則由剪力模數轉換。

Material-Property Example



a = 300 mm

b = 50 mm

t =20 mm

E_{cu}=120 Gpa

ν_{cu}=0.34

Determine its new length after application of the load.

$$\sigma_x = 800\text{MPa}, \quad \sigma_y = -500\text{MPa}, \quad \tau_{xy} = 0, \quad \sigma_z = 0$$

$$e_x = \frac{1}{E}[s_x - n(s_y + s_z)] = \frac{1}{120(10^3)}[800 - 0.34(-500 + 0)] = 0.00808$$

$$e_y = \frac{1}{E}[s_y - n(s_x + s_z)] = \frac{1}{120(10^3)}[-500 - 0.34(800 + 0)] = -0.00643$$

TOPIC

Mohr's Circle-Plane Strain

KEYPOINT

正向應變和應力間，可借由楊氏模數和柏松比轉換。剪應力和剪應變則由剪力模數轉換。

Material-Property Example

$s_x = 800\ \mathrm{MPa}, \quad s_y = -500\ \mathrm{MPa}$

$t_{xy} = 0, \quad s_z = 0$

$e_x = 0.00808 \quad e_y = -0.00643$

$\boldsymbol{a} = 300\ \mathrm{mm}$

$\boldsymbol{b} = 50\ \mathrm{mm}$

$\boldsymbol{t} = 20\ \mathrm{mm}$

$\boldsymbol{E_{cu}} = 120\ \mathrm{Gpa}$

$\boldsymbol{\nu_{cu}} = 0.34$

$$e_z = \frac{1}{E}[s_z - n(s_x + s_y)] = \frac{1}{120(10^3)}[0 - 0.34(800 - 500)] = -0.000850$$

$$a' = a + \varepsilon_x a = 300 + 0.00808(300) = 302.4\ \mathrm{mm}$$

$$b' = b + \varepsilon_y b = 50 + (-0.00643)(50) = 49.68\ \mathrm{mm}$$

$$t' = t + \varepsilon_z t = 20 + (-0.000850)(20) = 19.98\ \mathrm{mm}$$

TOPIC

Mohr's Circle-Plane Strain

KEYPOINT

正向應變和應力間，可借由楊氏模數和柏松比轉換。剪應力和剪應變則由剪力模數轉換。

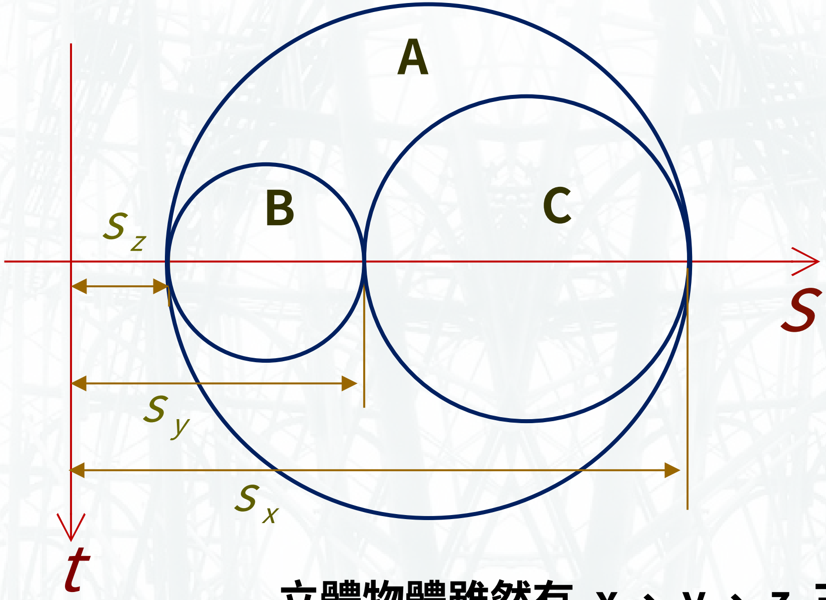
Triaxial stress

TOPIC

Mohr’s Circle-Plane Strain

KEYPOINT

正向應變和應力間，可借由楊氏模數和柏松比轉換。剪應力和剪應變則由剪力模數轉換。



Maximum *in-plane* shear stress

$$(\tau_{max})_z = \pm\frac{\sigma_x - \sigma_y}{2}$$

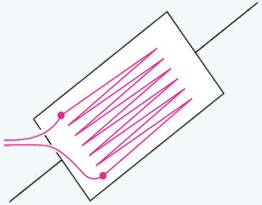
$$(\tau_{max})_x = \pm\frac{\sigma_y - \sigma_z}{2}$$

$$(\tau_{max})_y = \pm\frac{\sigma_x - \sigma_z}{2}$$

立體物體雖然有 x 、 y 、 z 三軸，但是仍可畫 A 、 B 、 C 莫耳圓來找應力相對關係。

TOPIC

版權聲明

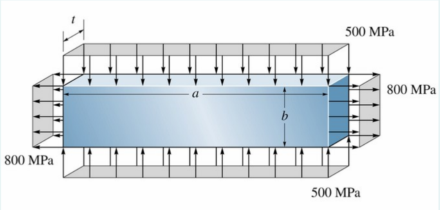
頁碼	作品	版權圖示	來源 / 作者
1			李星慧 2014 . 7. 高雄攝影作品，此圖經同意已進行藍階修改。 您如需利用本作品，請另行向權利人取得授權。
55			R.C. Hibbeler, “Mechanics of Materials”, 7th Ed. Pearson Prentice-Hall 2008 本作品依據著作權法第 46 、 52 、 65 條合理使用。

KEYPOINT

TOPIC

版權聲明

KEYPOINT

頁碼	作品	版權圖示	來源 / 作者
64			R.C. Hibbeler, “Mechanics of Materials”, 7th Ed. Pearson Prentice-Hall 2008 本作品依據著作權法第 46 、 52 、 65 條合理使用。