

Introduction to Computer Science

Lecture 9: COMPUTER 3D GRAPHICS

Tian-Li Yu

Taiwan Evolutionary Intelligence Laboratory (TEIL)
Department of Electrical Engineering
National Taiwan University

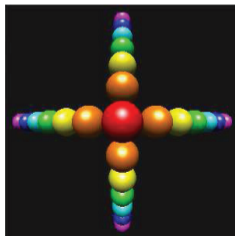
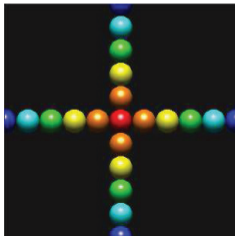
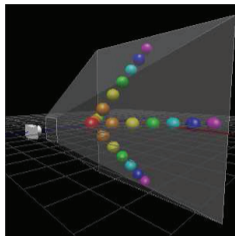
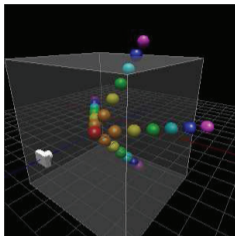
tianliyu@cc.ee.ntu.edu.tw

Slides made by Tian-Li Yu, Jie-Wei Wu, and Chu-Yu Hsu



【本著作除另有註明外，採取創用CC「姓名標示
—非商業性—相同方式分享」台灣3.0版授權釋出】

Parallel vs. Perspective Projection



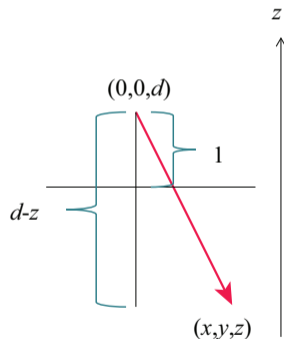
Simple Projection

- Suppose camera is fixed at $(0, 0, d)$.
- Projection plane is fixed at $z = d - 1$
- Usually keep z information
- Parallel projection

$$(x, y, z) \rightarrow (x, y, z)$$

- Prospective projection

$$(x, y, z) \rightarrow \left(\frac{x}{d-z}, \frac{y}{d-z}, z \right)$$



Translations & Rotations

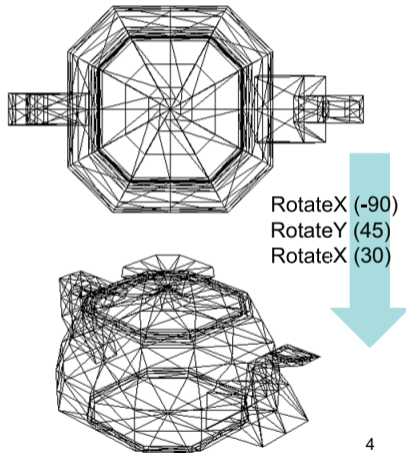
- Translations
 - Simply add a vector.

- Rotations

$$R_X(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta \\ 0 & \sin \theta & \cos \theta \end{bmatrix}$$

$$R_Y(\theta) = \begin{bmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{bmatrix}$$

$$R_Z(\theta) = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



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Modeling

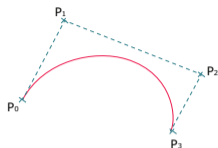
- Typically use triangles and quadrangles.
- Easy to compute the normal vector.

- How to increase resolution?
 - Spline and Bézier curves



Bézier Curves

- May be defined on different degree of polynomials.
- Here we introduce the cubic one with 4 control points:



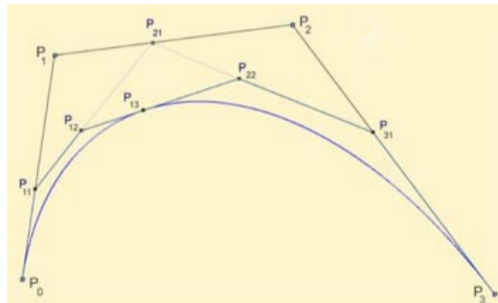
$$\begin{aligned} ((1-t) + t)^3 &= (1-t)^3 + 3t(1-t)^2 + 3t^2(1-t) + t^3 \\ P(t) &= (1-t)^3 P_0 + 3t(1-t)^2 P_1 + 3t^2(1-t) P_2 + t^3 P_3 \end{aligned}$$

Recursive Subdivision

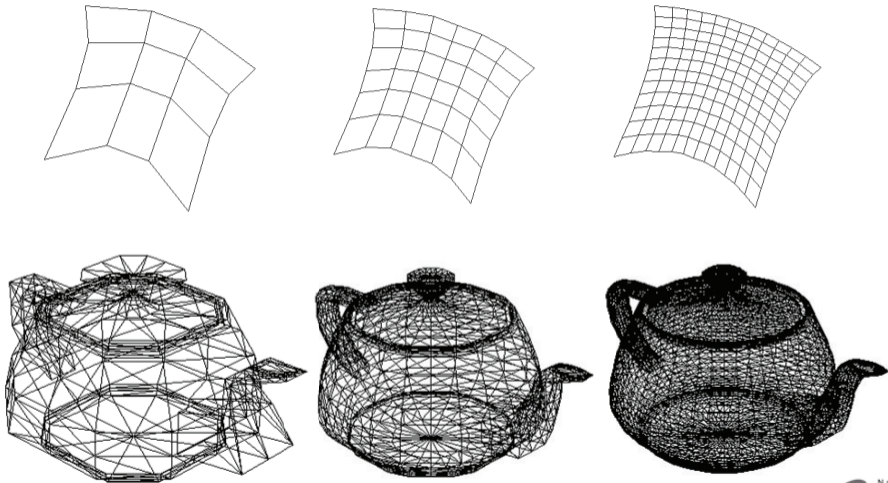
- A cubic Bézier curve can be subdivided into two cubic curves.

Given any t (0.4),

- $P_{11} = (1 - t)P_0 + tP_1$
- $P_{21} = (1 - t)P_1 + tP_2$
- $P_{31} = (1 - t)P_2 + tP_3$
- $P_{12} = (1 - t)P_{11} + tP_{21}$
- $P_{22} = (1 - t)P_{21} + tP_{31}$
- $P_{13} = (1 - t)P_{12} + tP_{22}$
- $P_0, P_{11}, P_{12}, P_{13}$ forms one curve.
- $P_{13}, P_{22}, P_{31}, P_3$ forms the other.



Recursive Subdivision

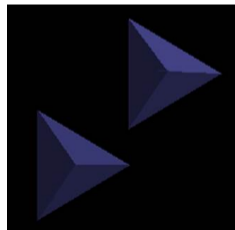
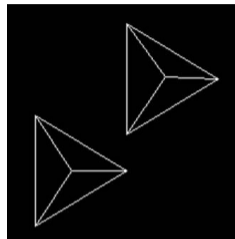


Lighting

- Ambient lighting
- Directional lighting
- Point lighting

$$I = I_a + \frac{(1 - I_a)}{1 + \alpha d + \beta d^2}$$

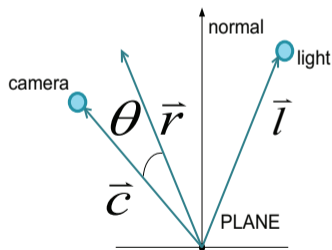
- d : distance between light source and target.
- I_a : intensity of ambient light.



Shading

\vec{n} : normal vector of plane.
 \vec{l} : normal vector of light.
 \vec{c} : normal vector of camera.
 \vec{r} : normal vector of reflection.

- Phong model
 - Ambient + Diffusion + Specular
- Diffusion: $\vec{n} \cdot \vec{l}$
- Specular: $(\cos \theta)^s$
 - Higher $s \rightarrow$ more mirror-like surface

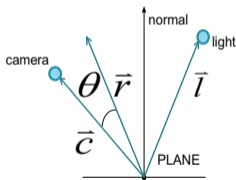


Specular

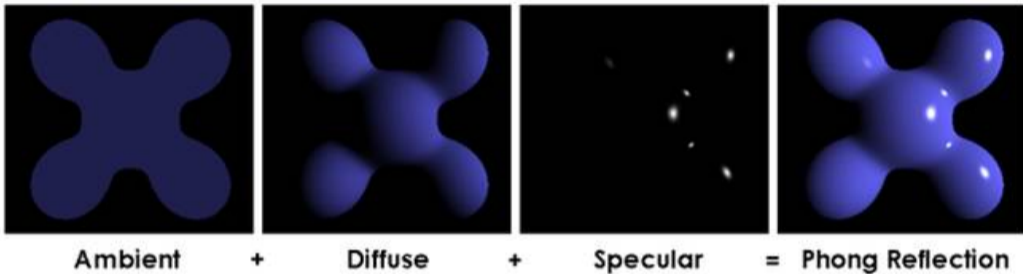
$$\vec{l} + \vec{r} = 2(\vec{l} \cdot \vec{n})\vec{n}$$

$$\vec{r} = 2(\vec{l} \cdot \vec{n})\vec{n} - \vec{l}$$

$$\cos \theta = \vec{r} \cdot \vec{c} = (2(\vec{l} \cdot \vec{n})\vec{n} - \vec{l}) \cdot \vec{c}$$



Put It All Together



$$C = I_a \cdot C_o + \frac{1 - I_a}{1 + \alpha d + \beta d^2} (k_d \cdot \vec{n} \cdot \vec{l} \cdot C_o + (1 - k_d) \cos^s \theta \cdot C_l)$$

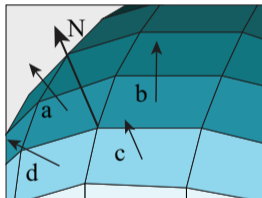
Summary of Parameters

Properties of light	
I_a	Ambient light intensity (0~1)
α, β	Degree of point lighting
C_l	Color of light
\vec{l}	Normal vector of light direction

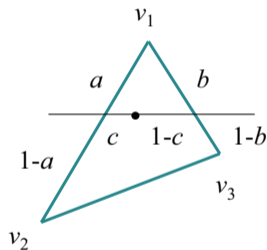
Properties of object	
k_d	Diffusion coefficient ($k_d = 1 - k_s$: specular coefficient)
s	Shininess: how mirror-like
C_o	Color of the object
\vec{n}	Normal vector of the plane

Flat, Gouraud, and Phong Shadings

- Flat: one triangle, one color.
- Gouraud: interpolation of vertices colors.
- Phong: interpolation of vertices normal vectors.



Vertex normal



$$(1 - c)((1 - a)v_1 + av_2) + c((1 - b)v_1 + bv_3)$$

Flat, Gouraud, and Phong Shadings



Flat

Gouraud

Phong



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