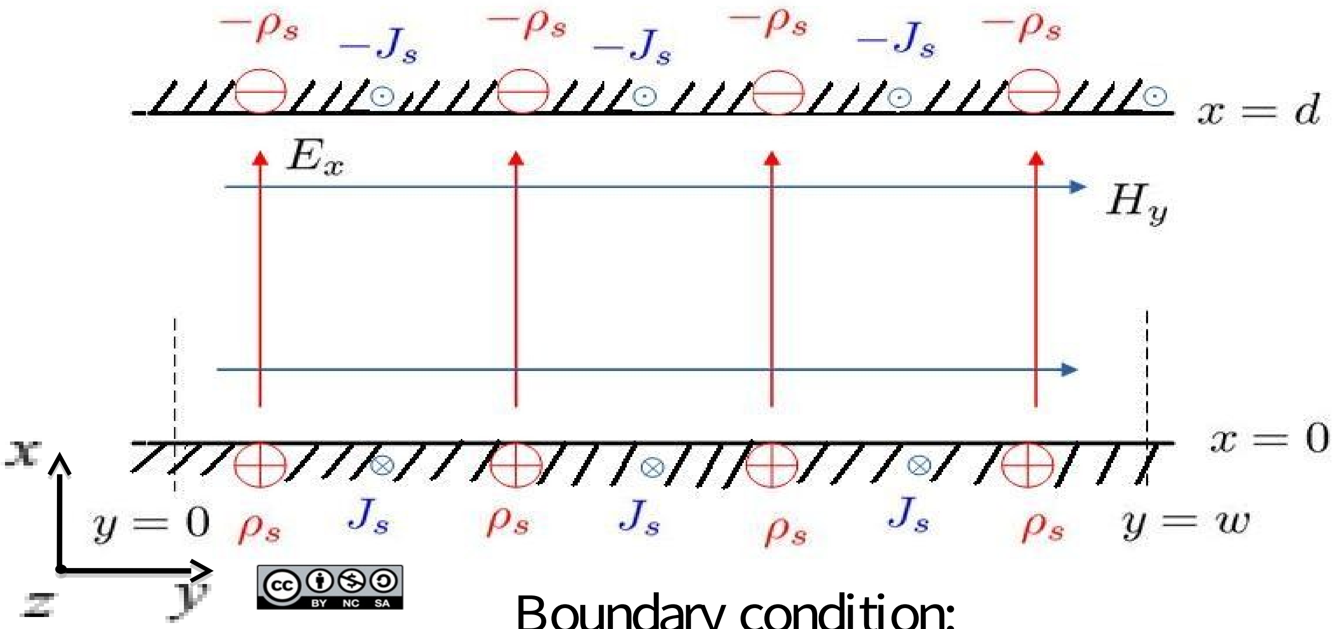
Electromagnetics II

Unit 1 ： Basics of Transmission Lines

Lecturer: Professor Jean-Fu Kiang

Text book: N. N. Rao, “Elements of Engineering Electromagnetics,” sixth ed., Pearson, 2004.

Parallel-Plate Waveguide



Boundary condition:

$$\rho_s = \hat{n} \cdot \overline{D} = \varepsilon E_x$$

$$\overline{J}_s = \hat{n} \times \overline{H} = \hat{z} H_y$$

Maxwell's Eqn. to Wave Eqn.

$$\frac{\partial E_x}{\partial z} = -\mu \frac{\partial H_y}{\partial t}$$

$$\frac{\partial H_y}{\partial z} = -\varepsilon \frac{\partial E_x}{\partial t}$$

$$\Rightarrow$$

$$\left(\frac{\partial^2}{\partial z^2} - \frac{1}{v_p^2} \frac{\partial^2}{\partial t^2} \right) E_x = 0$$

$$\left(\frac{\partial^2}{\partial z^2} - \frac{1}{v_p^2} \frac{\partial^2}{\partial t^2} \right) H_y = 0$$

Solution of Wave Equations

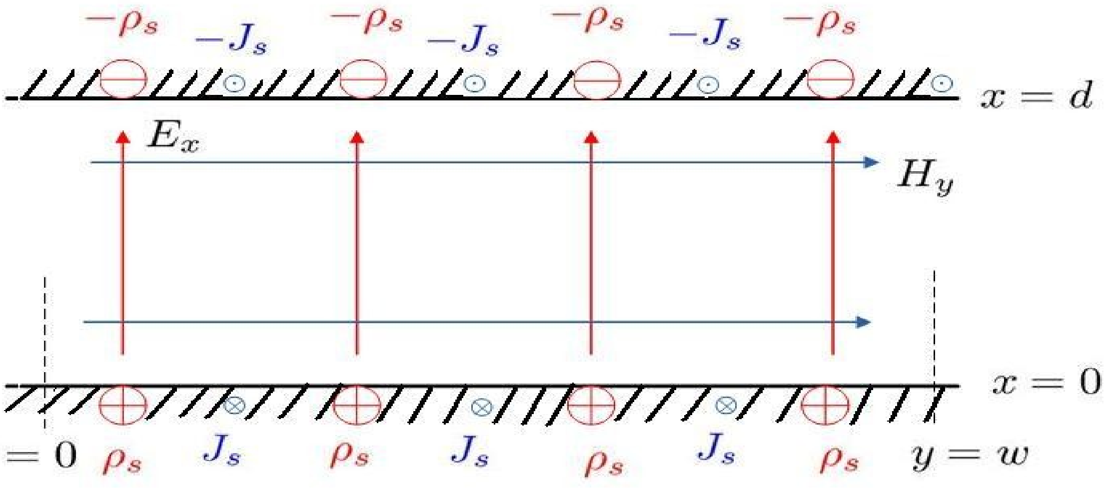
$$E_x(z,t) = f_+(t - z/v_p) + f_-(t + z/v_p)$$
$$H_y(z,t) = g_+(t - z/v_p) + g_-(t + z/v_p)$$

$f_+(t - z/v_p)$: propagate to $+z$ direction

$f_-(t + z/v_p)$: propagate to $-z$ direction

To maintain a fixed arguement, $(t - z/v_p)$, $z\uparrow$ as $t\uparrow$

Definition of *V* and *I*



$$\int_0^d E_x(z,t)\,dx = E_x(z,t)\,d = V(z,t) \qquad (1)$$

$$\int_0^w H_y(z,t)\,dy = H_y(z,t)\,w = I(z,t) \qquad (2)$$

Telegrapher's Equations

$$\frac{\partial(E_x d)}{\partial z} = \boxed{\frac{\partial V}{\partial z}} = -\mu \frac{d}{w}\frac{\partial(H_y w)}{\partial t} = -L\frac{\partial(H_y w)}{\partial t} = \boxed{-L\frac{\partial I}{\partial t}}$$

$$\frac{\partial(H_y w)}{\partial z} = \boxed{\frac{\partial I}{\partial z}} = -\varepsilon \frac{w}{d}\frac{\partial(E_x d)}{\partial t} = -C\frac{\partial(E_x d)}{\partial t} = \boxed{-C\frac{\partial V}{\partial t}}$$

$$\Rightarrow \begin{aligned} \frac{\partial V}{\partial z} &= -L\frac{\partial I}{\partial t} \qquad (3) \\ \frac{\partial I}{\partial z} &= -C\frac{\partial V}{\partial t} \qquad (4) \end{aligned}$$

Wave Equations of *V* and *I*

$$\frac{\partial}{\partial z}(3) \Rightarrow \quad \frac{\partial^2 V}{\partial z^2} = -L\frac{\partial}{\partial t}\frac{\partial I}{\partial t} = LC\frac{\partial^2 V}{\partial t^2}$$

where $LC = \left(\mu\frac{d}{w}\right)\left(\varepsilon\frac{w}{d}\right) = \mu\varepsilon = \frac{1}{v_p^2}$

$$\left(\frac{\partial^2}{\partial z^2} - \frac{1}{v_p^2}\frac{\partial^2}{\partial t^2}\right)V = 0$$

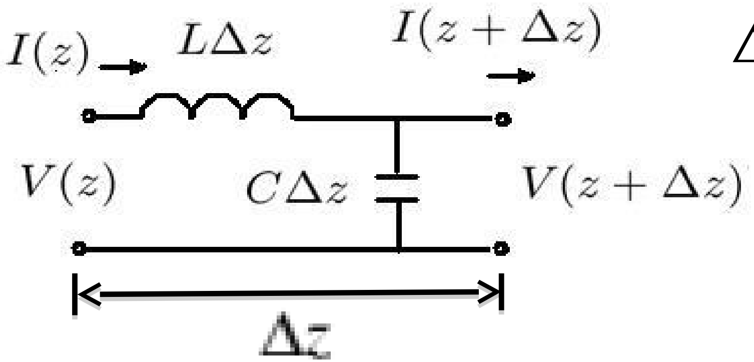
$$\left(\frac{\partial^2}{\partial z^2} - \frac{1}{v_p^2}\frac{\partial^2}{\partial t^2}\right)I = 0$$

Solution to Wave Equations of *V*, *I*

$$V(z,t)=f_+\left(t-z/v_p\right)+f_-\left(t+z/v_p\right)$$

$$I(z,t)=g_+\left(t-z/v_p\right)+g_-\left(t+z/v_p\right)$$

Equivalent Lumped-circuit of an Infinitesimal Parallel-plate Waveguide



$\Delta z \ll \lambda$

$$\text{KVL}: V(z) = L\Delta z \frac{\partial I(z,t)}{\partial t} + V(z+\Delta z,t)$$

$$V(z+\Delta z,t) \cong V + \frac{\partial V}{\partial z}\Delta z$$

$$\text{KCL}: I(z) = C\Delta z \frac{\partial V(z+\Delta z,t)}{\partial t} + I(z+\Delta z,t)$$

$$I(z+\Delta z,t) \cong I + \frac{\partial I}{\partial z}\Delta z$$

Voltage and Current waves

$$\boxed{V(z,t)=V_+\left(t-z/v_p\right)+V_-\left(t+z/v_p\right)}$$

$$(3)\rightarrow\frac{\partial I}{\partial t}=-\frac{1}{L}\frac{\partial V}{\partial z}=\frac{1}{Lv_p}\left[V_+'\left(t-z/v_p\right)-V_-'\left(t+z/v_p\right)\right]$$

$$\int\frac{\partial I}{\partial t}dt\rightarrow\boxed{I(z,t)=\frac{1}{Z_0}\left[V_+\left(t-z/v_p\right)-V_-\left(t+z/v_p\right)\right]}$$

where $\frac{1}{Lv_p}=\frac{1}{L}\sqrt{LC}=\sqrt{\frac{C}{L}}=\frac{1}{Z_0}$

Z_0: characteristic impedance (Ω)

Telegrapher's Eqn. in Frequency Domain

Time harmonic :

$$V(z,t)=\mathrm{Re}\left\{\underline{V}(z)e^{j\omega t}\right\}$$

$$I(z,t)=\mathrm{Re}\left\{\underline{I}(z)e^{j\omega t}\right\}$$

$$(3)\rightarrow\frac{\partial}{\partial z}\mathrm{Re}\left\{\underline{V}(z)e^{j\omega t}\right\}=-L\frac{\partial}{\partial t}\mathrm{Re}\left\{\underline{I}(z)e^{j\omega t}\right\},\forall t$$

$$\boxed{\frac{\partial}{\partial t}\rightarrow j\omega}$$

$$\Rightarrow\mathrm{Re}\left\{\frac{\partial\underline{V}(z)}{\partial z}e^{j\omega t}\right\}=-L\,\mathrm{Re}\left\{\underline{I}(z)\frac{\partial}{\partial t}e^{j\omega t}\right\},\forall t$$

$$\Rightarrow\boxed{\frac{\partial\underline{V}}{\partial z}=-j\omega L\underline{I}\qquad(5)}\ \boxed{\frac{\partial\underline{I}}{\partial z}=-j\omega C\underline{V}\qquad(6)}$$

Phasor Form of V and I

solve (5),(6)

$$\Rightarrow \boxed{\underline{V}(z) = V_+ e^{-jkz} + V_- e^{jkz}}$$

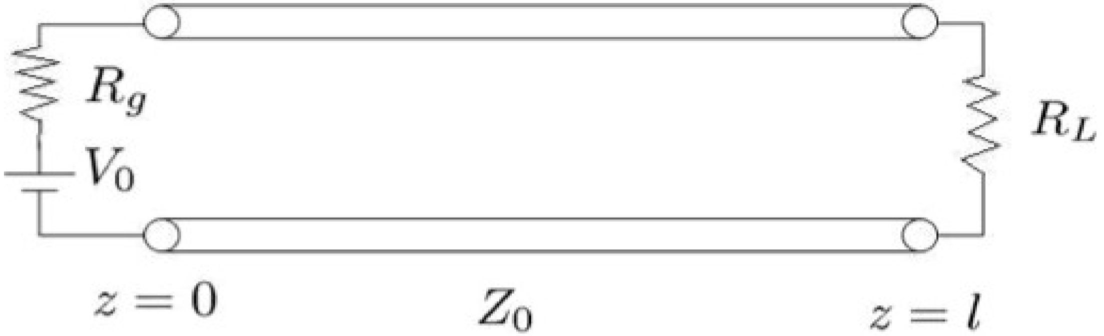
$$\Rightarrow V(z,t) = \mathrm{Re}\left\{\underline{V}(z) e^{j\omega t}\right\}$$

$$= |V_+| \cos(\omega t - kz + \phi_+) + |V_-| \cos(\omega t + kz + \phi_-)$$

$$(5) \rightarrow \underline{I} = -\frac{1}{j\omega L}\frac{\partial \underline{V}}{\partial z} = \frac{k}{\omega L} V_+ e^{-jkz} - \frac{k}{\omega L} V_- e^{jkz} = \frac{1}{Z_0}\left(V_+ e^{-jkz} - V_- e^{jkz}\right)$$

$$\boxed{\underline{I} = \frac{1}{Z_0}\left(V_+ e^{-jkz} - V_- e^{jkz}\right)}$$

Voltage and Current Waves along a Transmission Line

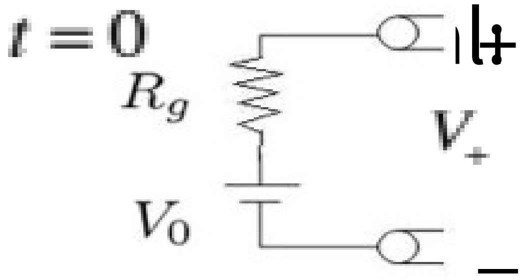


$$V(z,t)=V_+(t-z/v_p)+V_-(t+z/v_p)$$

$$I(z,t)=\frac{1}{Z_0}V_+(t-z/v_p)-\frac{1}{Z_0}V_-(t+z/v_p)$$

Voltage Reflection Coefficient by Transient-state Analysis (1)

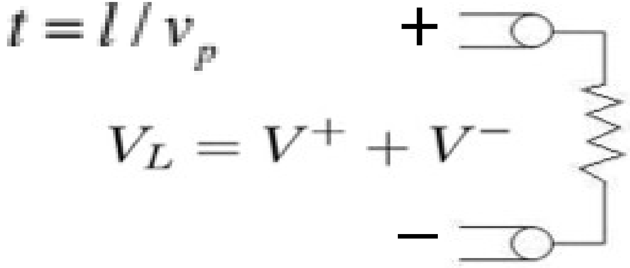
KVL at source



$$V_+ = R_g I_+ + V_+ = R_g \frac{V_+}{Z_0} + V_+$$

$$\Rightarrow V_+ = V_0 \frac{Z_0}{R_g + Z_0}$$

KVL at load



$$V_L = V^+ + V^-$$

$$V_L = R_L I_L$$

$$V_+ + V_- = R_L \left(\frac{V_+}{Z_0} - \frac{V_-}{Z_0} \right)$$

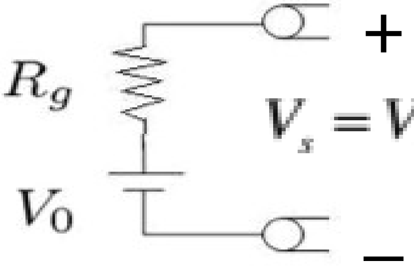
$$\Rightarrow V_- = V_+ \boxed{\frac{R_L - Z_0}{R_L + Z_0} = \Gamma_v} V_+$$

Voltage Reflection Coefficient by Transient-state Analysis (2)

$$V_0 = R_g I_s + V_s = R_g\left(\frac{V_s}{Z_0} - \frac{V_-}{Z_0} + \frac{V'_+}{Z_0}\right) + V_+ + V_- + V'_+$$

$$V'_+ = V_- \boxed{\left(\frac{R_g - Z_0}{R_g + Z_0}\right) = \Gamma_s} V_-$$

$$t = 2l / v_p$$



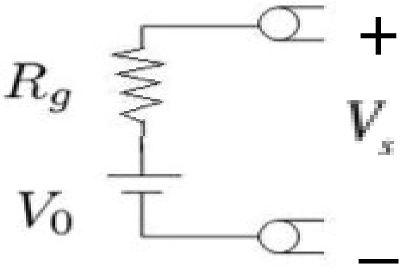
$$V_s = V_+ + V_- + V'_+$$

current reflection coefficient:

$$\Gamma_I = \frac{I_-}{I_+} = \frac{-V_- / Z_0}{V_+ / Z_0} = -\Gamma_v$$

Steady-state Voltage at Source Terminal

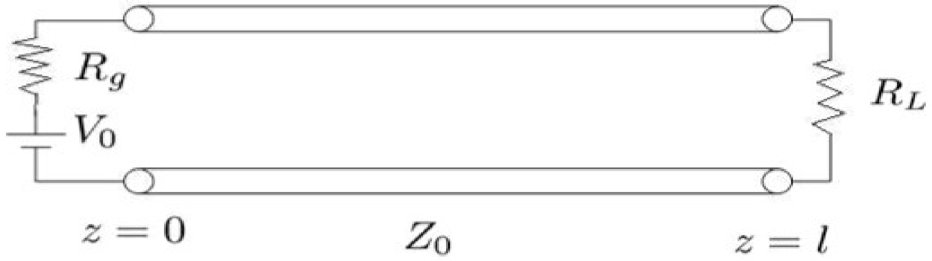
$t = \infty$

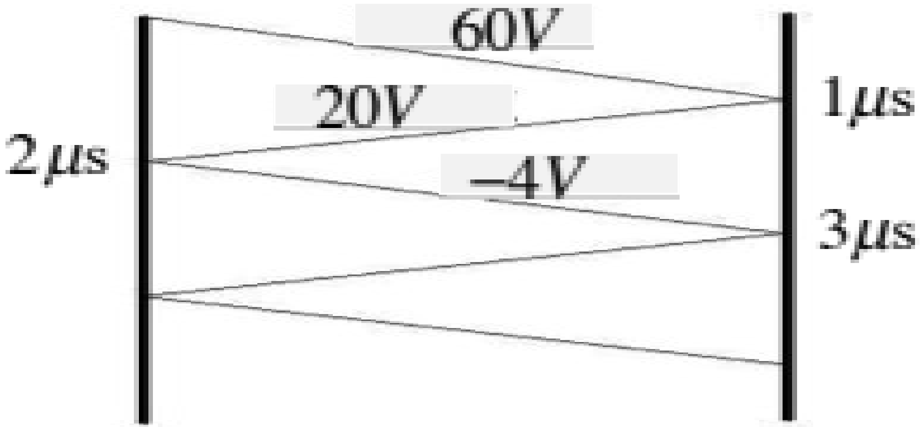


$$V_s = V_+ + V_- + V'_+ + \ldots$$

$$
\begin{aligned}
V(z=0, t=\infty) &= V_s = V_+ + V_- + V_+' + V_-' + \cdots \\
&= V_+ (1+\Gamma_L)(1 + \Gamma_L \Gamma_s + \Gamma_L^2 \Gamma_s^2 + \cdots) \\
&= V_+ (1+\Gamma_L) \frac{1}{1 - \Gamma_L \Gamma_s}
\end{aligned}
$$

Space-time Diagram





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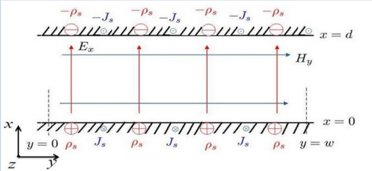
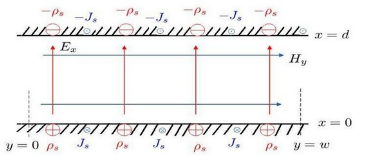
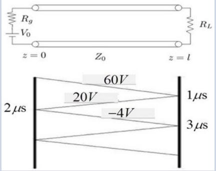
2		BY NC SA	Jean-Fu Kiang / National Taiwan University This work is licensed by Creative Commons Attribution-NonCommercial-ShareAlike 3.0 Taiwan
5		BY NC SA	Jean-Fu Kiang / National Taiwan University This work is licensed by Creative Commons Attribution-NonCommercial-ShareAlike 3.0 Taiwan
17		BY NC SA	Jean-Fu Kiang / National Taiwan University This work is licensed by Creative Commons Attribution-NonCommercial-ShareAlike 3.0 Taiwan