

Electromagnetics II

Unit 1 : Basics of Transmission Lines

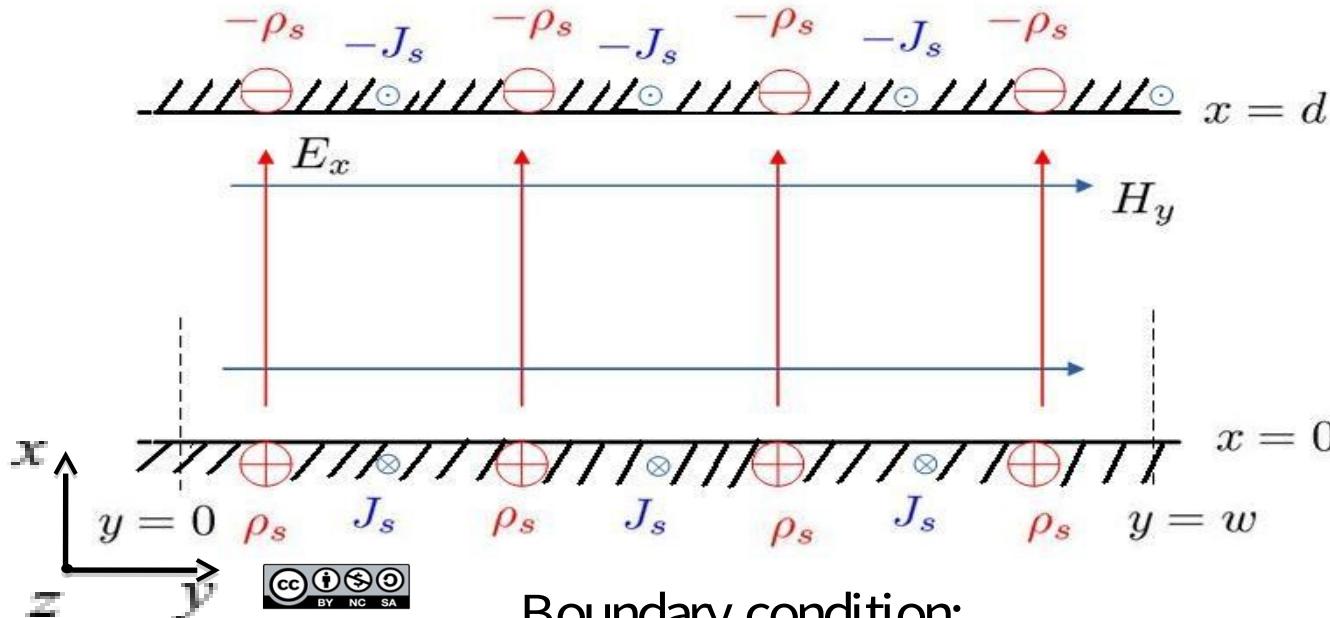
Lecturer: Professor Jean-Fu Kiang

Text book: N. N. Rao, “Elements of Engineering Electromagnetics,” sixth ed., Pearson, 2004.



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Parallel-Plate Waveguide



Boundary condition:

$$\rho_s = \hat{n} \cdot \bar{D} = \epsilon E_x$$

$$\bar{J}_s = \hat{n} \times \bar{H} = \hat{z} H_y$$

Maxwell's Eqn. to Wave Eqn.

$$\frac{\partial E_x}{\partial z} = -\mu \frac{\partial H_y}{\partial t}$$
$$\frac{\partial H_y}{\partial z} = -\epsilon \frac{\partial E_x}{\partial t}$$
$$\Rightarrow \left(\frac{\partial^2}{\partial z^2} - \frac{1}{v_p^2} \frac{\partial^2}{\partial t^2} \right) E_x = 0$$
$$\left(\frac{\partial^2}{\partial z^2} - \frac{1}{v_p^2} \frac{\partial^2}{\partial t^2} \right) H_y = 0$$

Solution of Wave Equations

$$E_x(z, t) = f_+(t - z/v_p) + f_-(t + z/v_p)$$

$$H_y(z, t) = g_+(t - z/v_p) + g_-(t + z/v_p)$$

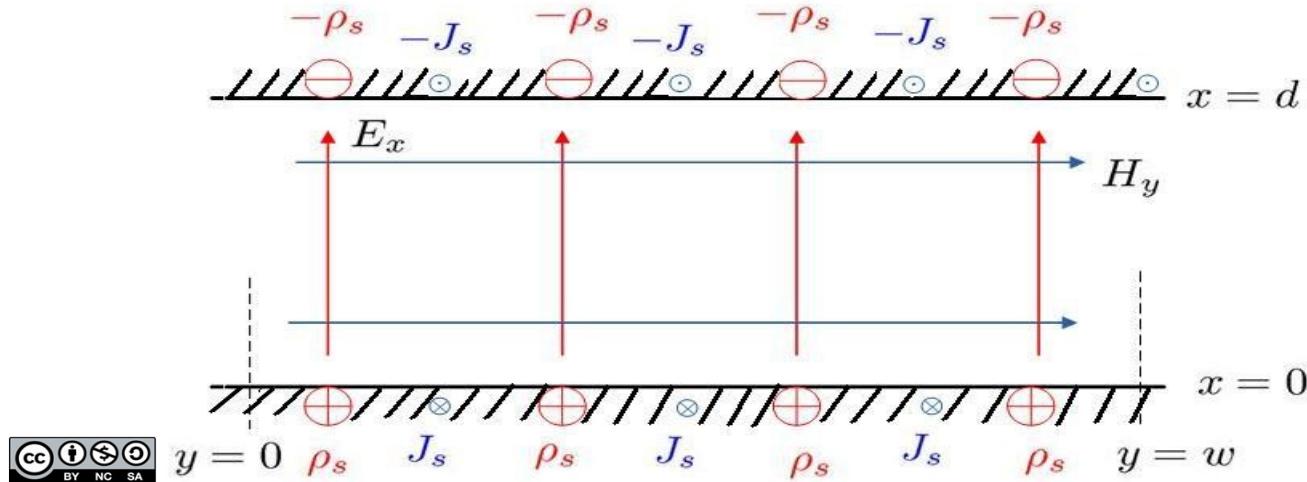
$f_+(t - z/v_p)$: propagate to $+z$ direction

$f_-(t + z/v_p)$: propagate to $-z$ direction

To maintain a fixed argument, $(t - z/v_p)$, $z \uparrow$ as $t \uparrow$



Definition of V and I



$$\int_0^d E_x(z, t) dx = E_x(z, t)d = V(z, t) \quad (1)$$

$$\int_0^w H_y(z, t) dy = H_y(z, t)w = I(z, t) \quad (2)$$

Telegrapher's Equations

$$\frac{\partial(E_x d)}{\partial z} = \boxed{\frac{\partial V}{\partial z}} = -\mu \frac{d}{w} \frac{\partial(H_y w)}{\partial t} = -L \frac{\partial(H_y w)}{\partial t} = \boxed{-L \frac{\partial I}{\partial t}}$$

$$\frac{\partial(H_y w)}{\partial z} = \boxed{\frac{\partial I}{\partial z}} = -\varepsilon \frac{w}{d} \frac{\partial(E_x d)}{\partial t} = -C \frac{\partial(E_x d)}{\partial t} = \boxed{-C \frac{\partial V}{\partial t}}$$

$$\frac{\partial V}{\partial z} = -L \frac{\partial I}{\partial t} \quad (3)$$

$$\Rightarrow \frac{\partial I}{\partial z} = -C \frac{\partial V}{\partial t} \quad (4)$$



Wave Equations of V and I

$$\frac{\partial}{\partial z}(3) \Rightarrow \frac{\partial^2 V}{\partial z^2} = -L \frac{\partial}{\partial t} \frac{\partial I}{\partial t} = LC \frac{\partial^2 V}{\partial t^2}$$

where $LC = \left(\mu \frac{d}{w} \right) \left(\varepsilon \frac{w}{d} \right) = \mu \varepsilon = \frac{1}{V_p^2}$

$$\left(\frac{\partial^2}{\partial z^2} - \frac{1}{V_p^2} \frac{\partial^2}{\partial t^2} \right) V = 0$$

$$\left(\frac{\partial^2}{\partial z^2} - \frac{1}{V_p^2} \frac{\partial^2}{\partial t^2} \right) I = 0$$

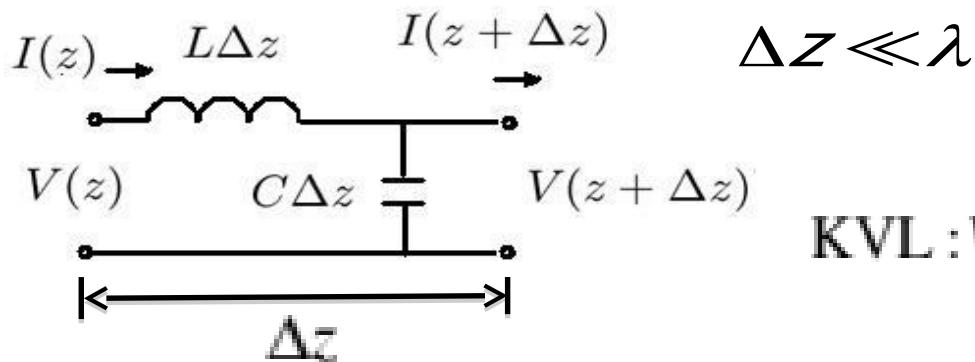


Solution to Wave Equations of V , I

$$V(z,t) = f_+(t - z/v_p) + f_-(t + z/v_p)$$

$$I(z,t) = g_+(t - z/v_p) + g_-(t + z/v_p)$$

Equivalent Lumped-circuit of an Infinitesimal Parallel-plate Waveguide



$$\Delta z \ll \lambda$$
$$\text{KVL : } V(z) = L\Delta z \frac{\partial I(z,t)}{\partial t} + V(z + \Delta z, t) \equiv V + \frac{\partial V}{\partial z} \Delta z$$

$$\text{KCL : } I(z) = C\Delta z \frac{\partial V(z + \Delta z, t)}{\partial t} + I(z + \Delta z, t) \equiv I + \frac{\partial I}{\partial z} \Delta z$$

Voltage and Current waves

$$V(z,t) = V_+(t - z/v_p) + V_-(t + z/v_p)$$

$$(3) \rightarrow \frac{\partial I}{\partial t} = -\frac{1}{L} \frac{\partial V}{\partial z} = \frac{1}{Lv_p} [V'_+(t - z/v_p) - V'_-(t + z/v_p)]$$

$$\int \frac{\partial I}{\partial t} dt \rightarrow I(z,t) = \frac{1}{Z_0} [V_+(t - z/v_p) - V_-(t + z/v_p)]$$

where $\frac{1}{Lv_p} = \frac{1}{L} \sqrt{LC} = \sqrt{\frac{C}{L}} = \frac{1}{Z_0}$ Z_0 : characteristic impedance (Ω)

Telegrapher's Eqn. in Frequency Domain

Time harmonic :

$$V(z,t) = \operatorname{Re}\{\underline{V}(z)e^{j\omega t}\}$$

$$I(z,t) = \operatorname{Re}\{\underline{I}(z)e^{j\omega t}\}$$

$$(3) \rightarrow \frac{\partial}{\partial z} \operatorname{Re}\{\underline{V}(z)e^{j\omega t}\} = -L \frac{\partial}{\partial t} \operatorname{Re}\{\underline{I}(z)e^{j\omega t}\}, \forall t$$

$$\Rightarrow \operatorname{Re}\left\{\frac{\partial \underline{V}(z)}{\partial z} e^{j\omega t}\right\} = -L \operatorname{Re}\left\{\underline{I}(z) \frac{\partial}{\partial t} e^{j\omega t}\right\}, \forall t$$

$$\boxed{\frac{\partial}{\partial t} \rightarrow j\omega}$$

$$\Rightarrow \boxed{\frac{\partial \underline{V}}{\partial z} = -j\omega L \underline{I}} \quad (5)$$

$$\boxed{\frac{\partial \underline{I}}{\partial z} = -j\omega C \underline{V}} \quad (6)$$



Phasor Form of V and I

solve (5),(6)

$$\Rightarrow \underline{V}(z) = V_+ e^{-jkz} + V_- e^{jkz}$$

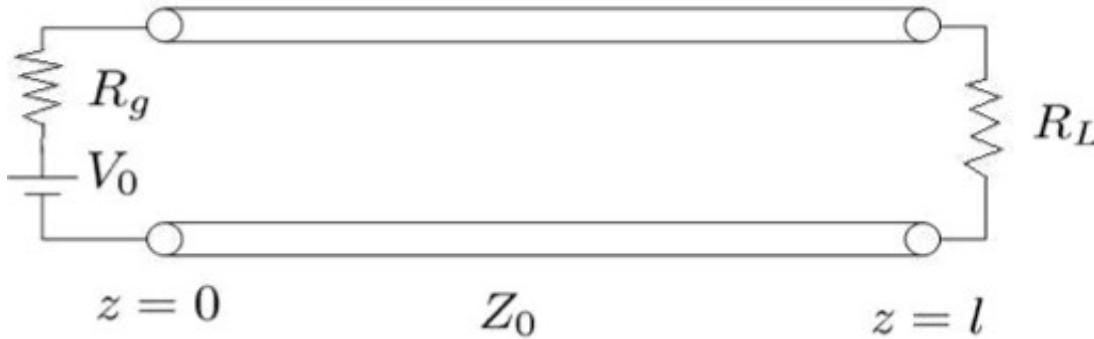
$$\begin{aligned}\Rightarrow V(z,t) &= \operatorname{Re}\{\underline{V}(z)e^{j\omega t}\} \\ &= |V_+| \cos(\omega t - kz + \phi_+) + |V_-| \cos(\omega t + kz + \phi_-)\end{aligned}$$

$$(5) \rightarrow I = -\frac{1}{j\omega L} \frac{\partial \underline{V}}{\partial z} = \frac{k}{\omega L} V_+ e^{-jkz} - \frac{k}{\omega L} V_- e^{jkz} = \frac{1}{Z_0} (V_+ e^{-jkz} - V_- e^{jkz})$$

$$I = \frac{1}{Z_0} (V_+ e^{-jkz} - V_- e^{jkz})$$



Voltage and Current Waves along a Transmission Line

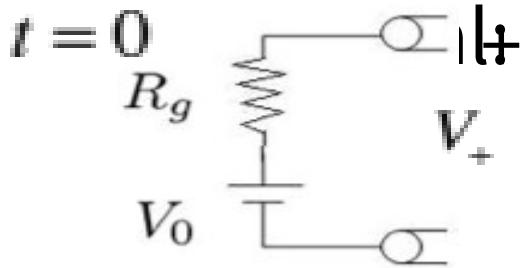


$$V(z,t) = V_+ \left(t - z/v_p \right) + V_- \left(t + z/v_p \right)$$

$$I(z,t) = \frac{1}{Z_0} V_+ \left(t - z/v_p \right) - \frac{1}{Z_0} V_- \left(t + z/v_p \right)$$

Voltage Reflection Coefficient by Transient-state Analysis (1)

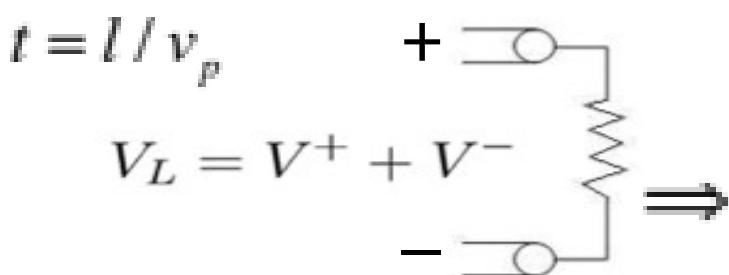
KVL at source



$$V_+ = R_g I_+ + V_+ = R_g \frac{V_+}{Z_0} + V_+$$

$$\Rightarrow V_+ = V_0 \frac{Z_0}{R_g + Z_0}$$

KVL at load



$$V_L = R_L I_L$$

$$V_+ + V_- = R_L \left(\frac{V_+}{Z_0} - \frac{V_-}{Z_0} \right)$$

$$V_- = V_+ \frac{\frac{R_L - Z_0}{R_L + Z_0}}{\frac{R_L + Z_0}{R_L - Z_0}} = \Gamma_v V_+$$

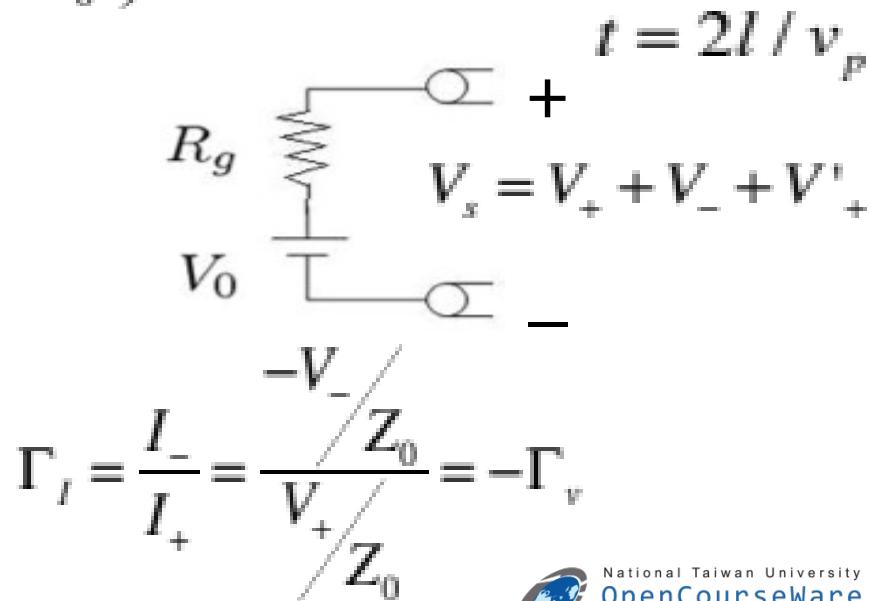


Voltage Reflection Coefficient by Transient-state Analysis (2)

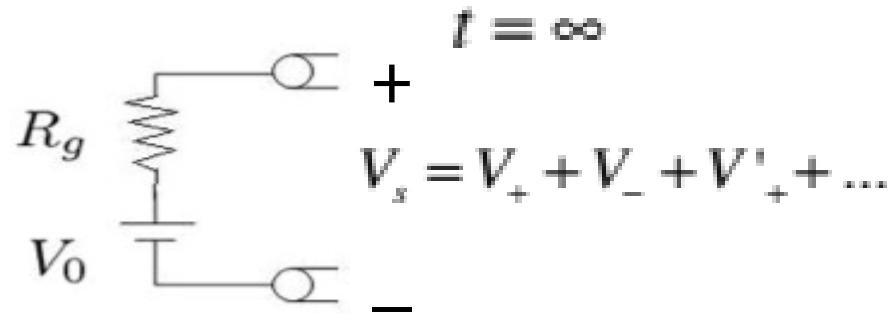
$$V_0 = R_g I_s + V_s = R_g \left(\frac{V_s}{Z_0} - \frac{V_-}{Z_0} + \frac{V'_+}{Z_0} \right) + V_+ + V_- + V'_+$$

$$V'_+ = V_- \left(\frac{R_g - Z_0}{R_g + Z_0} \right) = \Gamma_s V_-$$

current reflection coefficient:



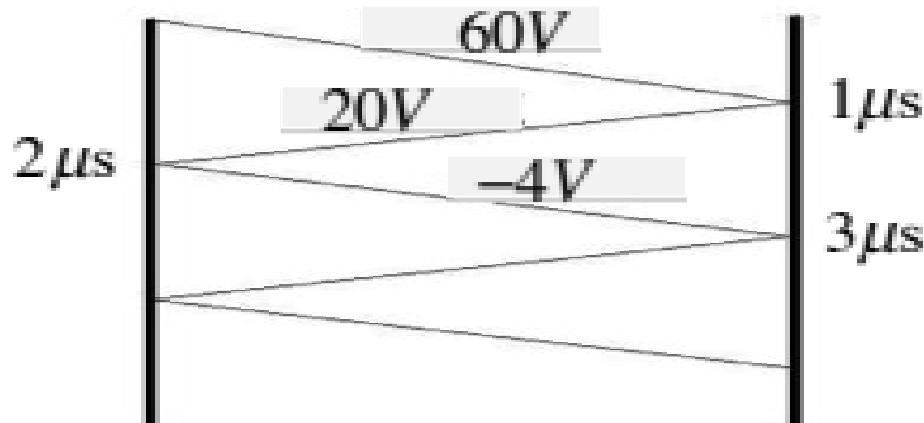
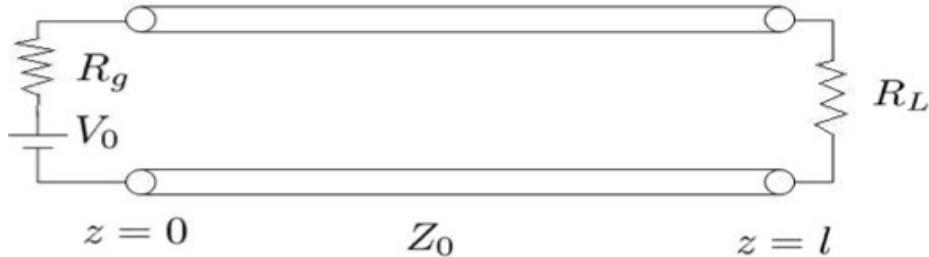
Steady-state Voltage at Source Terminal



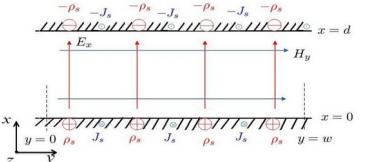
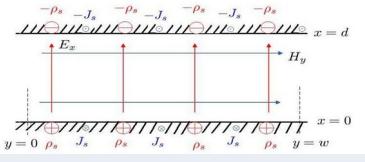
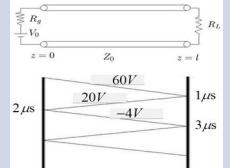
$$\begin{aligned}V(z=0, t=\infty) &= V_s = V_+ + V_- + V'_+ + V'_- + \dots \\&= V_+ (1 + \Gamma_L) (1 + \Gamma_L \Gamma_s + \Gamma_L^2 \Gamma_s^2 + \dots) \\&= V_+ (1 + \Gamma_L) \frac{1}{1 - \Gamma_L \Gamma_s}\end{aligned}$$



Space-time Diagram



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2	 <p>A diagram of a transmission line with length $x = d$ and width $y = w$. The line has alternating series impedances ρ_s and shunt admittances J_s. The top boundary conditions are $-\rho_s$, $-J_s$, $-\rho_s$, $-J_s$, $-\rho_s$, and $-J_s$. The bottom boundary conditions are ρ_s, J_s, ρ_s, J_s, ρ_s, and J_s. A coordinate system shows the line starting at $x=0$ and ending at $x=d$, with $y=0$ at the left and $y=w$ at the right.</p>		Jean-Fu Kiang / National Taiwan University This work is licensed by Creative Commons Attribution-NonCommercial-ShareAlike 3.0 Taiwan
5	 <p>A diagram of a transmission line with length $x = d$ and width $y = w$. The line has alternating series impedances ρ_s and shunt admittances J_s. The top boundary conditions are $-\rho_s$, $-J_s$, $-\rho_s$, $-J_s$, $-\rho_s$, and $-J_s$. The bottom boundary conditions are ρ_s, J_s, ρ_s, J_s, ρ_s, and J_s. A coordinate system shows the line starting at $x=0$ and ending at $x=d$, with $y=0$ at the left and $y=w$ at the right.</p>		Jean-Fu Kiang / National Taiwan University This work is licensed by Creative Commons Attribution-NonCommercial-ShareAlike 3.0 Taiwan
17	 <p>A circuit diagram showing a transmission line model. The line has length $z = l$ and characteristic impedance Z_0. The input voltage is V_0 and the load is R_L. The source is $2\mu s$ and the load is $1\mu s$. The transmission line has a time constant of $3\mu s$. The voltage across the line is $60V$ and the current is $20V$ and $-4V$.</p>		Jean-Fu Kiang / National Taiwan University This work is licensed by Creative Commons Attribution-NonCommercial-ShareAlike 3.0 Taiwan